Contents

Introduction

AWN (Algebra for Wireless Network protocols) is a process algebra that has been developed as a contribution to the formalization of wireless network protocols. It was proposed in 2012 [7] for the purpose of specifying WMN (Wireless Mesh Network) and MANET (Mobile Ad-hoc Network) protocols unambiguously and evaluating and validating them exhaustively. The ultimate goal of AWN is to reduce development time of (modifications of) WMN and MANET protocols and to increase their reliability and performance.
AWN's first use was in been demonstrated by modeling several interpretations of the AODV protocol [13], revealing, for instance, that not all of these interpretations guarantee loop freedom [23] or packet delivery [7]. One of the ambitions for AWN is that it will be possible to automatically model and verify AWN specifications with existing model checking toolsets via translations of AWN to input that these toolsets accept.
The graduation project of the author specifically involved the development of an automated translation from AWN to mCRL2 and to formally determine to which extent this translation is valid so that future users can be confident about the relation between their input and the results that mCRL2 produces. In this document, it will be shown that this relation can be developed to be a strong bisimulation.
AWN distinguishes itself from existing formalisms by disregarding packet loss as a possibility – that is, a message transmitted in AWN is received by all intended recipients within range. This abstraction allows the verification of the property of a network that a packet inserted by a client into the network is eventually delivered to the destination. AWN also features a conditional unicast operator and imperative 'mid-process' variable assignments.
In 2016, T-AWN was proposed [4], a *timed* process algebra reusing the existing AWN syntax. It was used to continue the investigation of the AODV protocol, and it was shown that *without* time abstraction the unambiguous interpretations of the AODV protocol always fail the loop freedom property.
Implementing one or more systematic translations from AWN to the input of a third-party model checking toolset would enable the automatic verification of network protocols, and thus faster protocol development. Such translations should be reliable, meaning that, at the very least, they should be theoretically valid under given premises. A manual translation from AWN to the UPPAAL model checker was published in 2012 [8] (this translation is currently being automated). Proving the validity of the translation resulted in difficulties, however, due to incomplete UPPAAL semantics.
The graduation project of the author focuses on a translation from AWN to mCRL2, a generic process algebra with an accompanying modeling and verification toolset [12], instead. mCRL2 (*milli* Common Representation Language) is the successor of μCRL (*micro* Common Representation Language), a specification language from 1990 designed using a minimalistic approach to replace a previously constructed common representation language so 'monstrous' that it did not facilitate further theory and tool development [11]. However, μCRL lacked features that were frequently used in specifications of protocols and concurrent systems. Eventually mCRL2 was proposed to make μCRL more applicable in practice.
The project will involve a formal specification of the AWN-to-mCRL2 translation and a proof of its validity. Because AWN and mCRL2 are both process algebras, proving the validity of the translation from one to the other is expected to be more straightforward than the translation from AWN to UPPAAL. It is also likely that an automatic translation will be implemented in the course of the project, possibly by making use of model-driven engineering as the main technique. Tasks to fill any remaining time will be determined at a later date (this report describes several possibilities).

Part 1

Motivation

1 Why mCRL2?

In the first stage of the research mCRL2 was chosen as the target model checking toolset for the translation from AWN. This was done by choosing several criteria that the target toolset should satisfy. These criteria were used to make a selection of model checking toolsets found in internet databases [17] [18] [24]. In general, this selection process has suffered from a time constraint that was self-imposed to prevent the comparison of existing toolsets from being taken further than what is beneficial to this graduation project. Searching for unambiguous information about a certain model checking toolset was therefore stopped after approximately 1.5 hours. As a consequence, information might have been missed, and a toolset might have been dismissed unjustly.
Furthermore, there are multiple model checking toolsets that are based on their own particular algebra or logic. A comparison of these toolsets would require much more in-depth study of and experience with these toolsets, which would consume valuable project time. It is therefore possible that relevant information was misunderstood or incorrectly considered to be ambiguous.
Finally, it should be noted that the databases where model checking toolsets were found cannot be expected to contain *all* model checking toolsets, and it is an assumption made by the author that the toolsets that were found are at least representative of the state-of-the-art functionality.
The chosen criteria for the model checking toolset to be targeted by the translation from AWN are these:

1. A toolset should have clear support for generic model checking, and not be specifically aimed at, for instance, code verification or probabilistic model checking.
2. A toolset should have accessible, complete, and up-to-date formal semantics. This is required to prove the validity of the translation later in the research.
3. A toolset should have sufficient online documentation, including examples, tutorials, and possibly active user forums. This makes the toolset easier to learn, which facilitates more advanced use of that toolset as a back end for AWN.
4. A toolset must have been updated since 2012 (in the previous 5 years). This criteria aims to exclude tools that are no longer being developed and improved without having to rely on that information being publicly available (which it may not be), and to exclude tools with very slow release cycles.

Toolset	Failed criteria
ECW	4
IVy	6
Mobility Workbench	4
PEP	3, 6
Spot 2.0	2
TAPAs	4
TLA+	6
UCLID 3	4
UPPAAL	(An AWN-to-UPPAAL translation is already in development.)
ZING	2

Table 1.1: *Toolsets that were rejected and why.*

Toolset	Modelling language(s)	Reference
ARC	AltaRica	[1]
CADP	LOTOS, FSP, LOTOS NT	[9]
FDR	CSPM	[10]
LTSmin	Promela, μCRL, mCRL2	[3]
mCRL2	mCRL2	[6]
nuXmv	SMV	[5]
SPIN	Promela	[14]

Table 1.2: *Selected toolsets.*

5. A toolset should have a text-based, sufficiently expressive modeling language, so that the result of a translation is easily readable by users and manual adjustments can be made if desired. This implies the presence of a framework for modeling concurrent systems, for example, but also more basic features, such as data types, functions, arithmetical operators, and so on.
6. A toolset should have a sufficiently expressive property language, so that a maximum number of different types of properties of a protocol can be analyzed. Toolsets with more expressive property languages are preferred over those with less expressive ones.

Criteria 1 has been used as an initial filter to reduce the number of model checking toolsets. Table 1.1 lists the toolsets that failed one or more of the other criteria. From the toolsets that remained, listed in Table 1.2, mCRL2 was chosen because of prior experience with mCRL2 of the author as well as the similarity between AWN and mCRL2 (they are both process algebra languages). Additionally, there exists a LTSmin [3] back-end for mCRL2, so translating to mCRL2 yields two supporting tools 'for the price of one'.

2 Why MDE?

Comparison of MDE and Scala framework.
Since the graduation project will involve a translation with an accompanying validity proof, it is likely that the implementation of the graduation project will become the definitive framework for future translations. There is a strong argument to be made to use the existing Scala framework for AWN translations (see Section **??**) for the graduation project by extending it, namely that it already is a work in progress and that using the design in Figure **??** would mean that part of that work would be wasted.
There are other considerations, as well. The Scala framework only requires knowledge of 1 general-purpose language (Scala) and 1 syntax language, whereas the MDE approach requires developers to know 1 general-purpose language (Java or another language supported by MDE tools) and at least 4 DSLs (there exist different DSLs for the same purpose in MDE). Due to its multitude of shorthands and features, Scala is more versatile than Java, although this comes at the cost of a steeper learning curve and reduced readability.
A prominent advantage of MDE is that there exist metametamodel standards that are supported by existing frameworks – in other words, MDE applications that use the same metametamodel standard can easily be modified to exchange models, even if their underlying MDE implementations are entirely different. This is particularly useful for the development of front-ends and back-ends: a metamodel essentially becomes a partial application programming interface (API). For AWN translations, this means that a single translation file could be sufficient to develop a translation to a model checking language if there exists a metamodel for that language.
Another advantage of MDE is that it automatically sets a high standard for the degree at which the different aspects of an implementation are separated, similar to *aspect-oriented techniques* [16]. The Scala framework, on the other hand, is more prone to situations in which multiple concerns are addressed in one place, reducing code maintainability.
Because it is unclear how these advantages and disadvantages should be weighed, the design of the AWN translation framework will be decided in the course of the graduation project itself (see Section **??**).

Part 2

Formal translation

1 AWN semantics

This section will give an overview of the semantics of AWN. The corresponding inference rules can be found in Tables 2.1 and 2.2. The labels that identify the inference rules show a number in between brackets which refers to the table in [7] from which they were copied. For a full description of the semantics of AWN, consult that document.
The semantics of AWN are divided into four 'levels':

- The *sequential process level*. The semantics of this level describe how the decision flow of a single process of a protocol is specified, including guards, variable assignments, and local broadcasts.
- The *parallel process level* combines multiple sequential processes into a single parallel process so that they can run on the same node. Combined sequential processes can only communicate in one direction.
- The *node level* gives parallel processes their address and the set of addresses of nodes that are within their transmission range. It also adds network behavior such as connecting and disconnecting to parallel processes.
- The *network level* determines which behavior is ultimately allowed to occur: a node that tries to transmit a message, for example, will only succeed if a recipient actively cooperates.

1.1 Sequential process level

The decision flow of a protocol is determined by a composition of sequential processes. Sequential processes have a signature $X(\mathtt{var}_1, \ldots, \mathtt{var}_n)$ consisting of a name X and a number of parameters $\mathtt{var}_i$, and their behavior is specified via a sequential process expression SP with the following grammar:

$$\begin{aligned} \mathrm{SP} ::= \; & [\phi]\,\mathrm{SP} \;\mid\; [\![\mathtt{var} := \mathit{exp}]\!]\,\mathrm{SP} \\ & \mid \mathbf{broadcast}(\mathit{msg}).\mathrm{SP} \;\mid\; \mathbf{groupcast}(\mathit{dests}, \mathit{msg}).\mathrm{SP} \;\mid\; \mathbf{unicast}(\mathit{dest}, \mathit{msg}).\mathrm{SP} \blacktriangleright \mathrm{SP} \\ & \mid \mathbf{send}(\mathit{msg}).\mathrm{SP} \;\mid\; \mathbf{deliver}(\mathit{data}).\mathrm{SP} \;\mid\; \mathbf{receive}(\mathtt{msg}).\mathrm{SP} \\ & \mid \mathrm{X}(\mathit{exp}_1, \ldots, \mathit{exp}_n) \end{aligned}$$

Clearly, sequential processes have many possibilities to determine the behavior of a protocol: using guards $[\phi]\,\mathrm{SP}$, they can elect to perform certain actions only under particular circumstances; variable assignment $[\![\mathtt{var} := \mathit{exp}]\!]\,\mathrm{SP}$ allows the contents of the internal data structure of the sequential process to be changed; they can perform a **broadcast**, **groupcast**, **unicast**, **send**, **deliver**, or **receive** action, which allow processes to exchange messages in specific ways; they can let another sequential process $\mathrm{X}(\mathit{exp}_1, \ldots, \mathit{exp}_n)$ determine their subsequent behavior; and they can choose non-deterministically between multiple of these possibilities via the $+$ operator.
The different types of message exchanges have distinct purposes: **broadcast** is used to send a message to *all* nodes in the network that are within range; **groupcast** does the same as long as nodes are in a specified subset; **unicast** allows a message to be sent to a specified node, continuing with its first branch p if that node is within range or continuing with a $\neg$**unicast** action and its second branch q if the transmission failed; and **send** passes a message to another sequential process running on the same node (see Section 1.2).

$$\frac{}{\xi, \mathbf{broadcast}(ms).\mathrm{p} \xrightarrow{\mathbf{broadcast}(\xi(ms))} \xi, \mathrm{p}}\ \textsc{Broadcast (T1)}$$

$$\frac{}{\xi, \mathbf{groupcast}(dests, ms).\mathrm{p} \xrightarrow{\mathbf{groupcast}(\xi(dests),\xi(ms))} \xi, \mathrm{p}}\ \textsc{Groupcast (T1)}$$

$$\frac{}{\xi, \mathbf{unicast}(dest, ms).\mathrm{p} \blacktriangleright \mathrm{q} \xrightarrow{\mathbf{unicast}(\xi(dest),\xi(ms))} \xi, \mathrm{p}}\ \textsc{Unicast (T1-1)}$$

$$\frac{}{\xi, \mathbf{unicast}(dest, ms).\mathrm{p} \blacktriangleright \mathrm{q} \xrightarrow{\neg\mathbf{unicast}(\xi(dest),\xi(ms))} \xi, \mathrm{q}}\ \textsc{Unicast (T1-2)}$$

$$\frac{}{\xi, \mathbf{send}(ms).\mathrm{p} \xrightarrow{\mathbf{send}(\xi(ms))} \xi, \mathrm{p}}\ \textsc{Send (T1)}$$

$$\frac{}{\xi, \mathbf{deliver}(data).\mathrm{p} \xrightarrow{\mathbf{deliver}(\xi(data))} \xi, \mathrm{p}}\ \textsc{Deliver (T1)}$$

$$\forall m \in \mathrm{MSG}\ \frac{}{\xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p} \xrightarrow{\mathbf{receive}(m)} \xi[\mathtt{msg} := m], \mathrm{p}}\ \textsc{Receive (T1)}$$

$$\frac{}{\xi, [\![\mathtt{var} := exp]\!]\mathrm{p} \xrightarrow{\tau} \xi[\mathtt{var} := \xi(exp)], \mathrm{p}}\ \textsc{Assignment (T1)}$$

$$\forall a \in \mathrm{Act}\ \frac{\emptyset[\mathtt{var}_i := \xi(exp_i)]_{i=1}^{n}, \mathrm{p} \xrightarrow{a} \zeta, \mathrm{p}' \qquad \mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}}{\xi, \mathrm{X}(exp_1, \cdots, exp_n) \xrightarrow{a} \zeta, \mathrm{p}'}\ \textsc{Recursion (T1)}$$

$$\forall a \in \mathrm{Act}\ \frac{\xi, \mathrm{p} \xrightarrow{a} \zeta, \mathrm{p}'}{\xi, \mathrm{p} + \mathrm{q} \xrightarrow{a} \zeta, \mathrm{p}'}\ \textsc{Choice (T1-1)} \qquad \forall a \in \mathrm{Act}\ \frac{\xi, \mathrm{q} \xrightarrow{a} \zeta, \mathrm{q}'}{\xi, \mathrm{p} + \mathrm{q} \xrightarrow{a} \zeta, \mathrm{q}'}\ \textsc{Choice (T1-2)}$$

$$\zeta(\phi) = \mathit{true} \wedge \{\mathtt{q}_1, \cdots, \mathtt{q}_n\} = \mathrm{Fv}(\phi) \setminus \mathrm{dom}(\xi)\ \frac{\zeta = \xi[\mathtt{q}_1 := e_1, \cdots, \mathtt{q}_n := e_n]}{\xi, [\phi]\mathrm{p} \xrightarrow{\tau} \zeta, \mathrm{p}}\ \textsc{Guard (T1)}$$

$$\forall a \neq \mathbf{receive}(m)\ \frac{\mathrm{P} \xrightarrow{a} \mathrm{P}'}{\mathrm{P} \lll \mathrm{Q} \xrightarrow{a} \mathrm{P}' \lll \mathrm{Q}}\ \textsc{Parallel (T2-1)} \qquad \forall a \neq \mathbf{send}(m)\ \frac{\mathrm{Q} \xrightarrow{a} \mathrm{Q}'}{\mathrm{P} \lll \mathrm{Q} \xrightarrow{a} \mathrm{P} \lll \mathrm{Q}'}\ \textsc{Parallel (T2-2)}$$

$$\forall m \in \mathrm{MSG}\ \frac{\mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}' \qquad \mathrm{Q} \xrightarrow{\mathbf{send}(m)} \mathrm{Q}'}{\mathrm{P} \lll \mathrm{Q} \xrightarrow{\tau} \mathrm{P}' \lll \mathrm{Q}'}\ \textsc{Parallel (T2-3)}$$

$$\frac{\mathrm{P} \xrightarrow{\mathbf{broadcast}(m)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{R:*\mathbf{cast}(m)} ip : \mathrm{P}' : R}\ \textsc{Broadcast (T3)} \qquad \frac{\mathrm{P} \xrightarrow{\mathbf{groupcast}(D,m)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{R\cap D:*\mathbf{cast}(m)} ip : \mathrm{P}' : R}\ \textsc{Groupcast (T3)}$$

$$\frac{\mathrm{P} \xrightarrow{\mathbf{deliver}(d)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{ip:\mathbf{deliver}(d)} ip : \mathrm{P}' : R}\ \textsc{Deliver (T3)} \qquad \frac{\mathrm{P} \xrightarrow{\tau} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{\tau} ip : \mathrm{P}' : R}\ \textsc{Internal (T3)}$$

$$\frac{\mathrm{P} \xrightarrow{\mathbf{unicast}(dip,m)} \mathrm{P}' \qquad dip \in R}{ip : \mathrm{P} : R \xrightarrow{\{dip\}:*\mathbf{cast}(m)} ip : \mathrm{P}' : R}\ \textsc{Unicast (T3-1)} \qquad \frac{\mathrm{P} \xrightarrow{\neg\mathbf{unicast}(dip,m)} \mathrm{P}' \qquad dip \notin R}{ip : \mathrm{P} : R \xrightarrow{\tau} ip : \mathrm{P}' : R}\ \textsc{Unicast (T3-2)}$$

$$\frac{\mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{\{ip\}\neg\emptyset:\mathbf{arrive}(m)} ip : \mathrm{P}' : R}\ \textsc{Arrive (T3-1)} \qquad \frac{}{ip : \mathrm{P} : R \xrightarrow{\emptyset\neg\{ip\}:\mathbf{arrive}(m)} ip : \mathrm{P} : R}\ \textsc{Arrive (T3-2)}$$

Table 2.1: *AWN inference rules (1/2)*

The **receive** action performs the complementary action: it intercepts a message (regardless of whether it originated from a **broadcast**, **groupcast**, **unicast**, or **send** action) and stores it in a specified variable. Messages (or rather, the relevant data that they contain) can exit the network via the **deliver** action.
Inference rules Broadcast (T1) to Guard (T1) in Table 2.1 give the semantics of sequential process expressions ξ, p in AWN (in these expressions, ξ is a variable-to-value mapping and p the current sequential process expression).

Interpretation of guards

Guards in AWN use the syntax $[\phi]\ p$. The rule in Table 2.1 that allows guards to be constructed (Guard (T1)) makes use of the notation $\xi \xrightarrow{\phi} \zeta$. Informally, this syntax is defined to mean that the variable-to-value mapping ζ extends variable-to-value mapping ξ with new mappings for variables unmapped in ξ in such a way that ϕ under ζ evaluates to *true*. This allows AWN to set variables in guards using a type of *pattern matching*. For example, let `ip` and `data` be unmapped when the following expression is executed:

$$\textbf{receive}(\texttt{msg}).[\texttt{msg} = \texttt{Message}(\texttt{ip}, \texttt{data})]\ p$$

The execution will reach p if and only if a message is received that conforms to the structure $\texttt{Message}(ip, data)$, after which p is executed with $\xi(\texttt{ip}) := ip$ and $\xi(\texttt{data}) := data$.

For the correctness proof of the translation, a formal interpretation of $\xi \xrightarrow{\phi} \zeta$ is needed. To this end, Guard (T1) uses the following side condition:

$$\zeta(\phi) = \mathit{true} \wedge \{\mathtt{q}_1, \cdots, \mathtt{q}_n\} = \mathrm{Fv}(\phi) \setminus \textsc{dom}(\xi)$$

where $\mathrm{Fv}(\phi)$ are the free variables in ϕ and $\textsc{dom}(\xi) = \{\ \mathtt{x} \mid \mathtt{x} := \mathtt{y} \in \xi\ \}$.

1.2 Parallel process level

Sequential processes are combined into a single parallel process according to the following grammar:

$$\mathrm{PP} ::= \ \xi, \mathrm{SP} \quad | \quad \mathrm{PP} \parallel \mathrm{PP}$$

The sequential processes form a pipeline for messages: processes can use the **send** action to send a message to the process to their left, where the **receive** action can be used to receive it. Only the rightmost sequential process can use **receive** to listen for messages from other nodes in the network. See inference rules Parallel (T2-1) to Parallel (T2-3) in Table 2.1 for the semantics.

1.3 Node level

The rules for the node level, Broadcast (T3) to Disconnect (T3-3), can be found in Tables 2.1 and 2.2. These rules add network behavior to parallel processes: they allow them to connect, to disconnect, to deliver data objects, and they rewrite actions such as **broadcast**(m) and **unicast**(d, m) in terms of the current state of the network: **broadcast**(m) will be converted to R : **starcast**(m), where R is the set of addresses of all nodes within range, whereas **unicast**(d, m) is converted to the action $\{d\}$: **starcast**(m) because only the node with address d should be a recipient.
The inference rules for the node level make use of the $ip : \mathrm{PP} : R$ notation to consistently access the part of the network state relevant to a node. In this notation PP is a syntactic parallel process expression, ip is the address of the node that runs PP (as a semantic value), and R is the set of addresses of nodes that are within range of that node (also as a semantic value).

$$\frac{}{ip : \mathrm{P} : R \xrightarrow{\mathbf{connect}(ip, ip')} ip : \mathrm{P} : R \cup \{ip'\}}\ \textsc{Connect}\ (\text{T3-1})$$

$$\frac{}{ip : \mathrm{P} : R \xrightarrow{\mathbf{disconnect}(ip, ip')} ip : \mathrm{P} : R \setminus \{ip'\}}\ \textsc{Disconnect}\ (\text{T3-1})$$

$$\frac{}{ip : \mathrm{P} : R \xrightarrow{\mathbf{connect}(ip', ip)} ip : \mathrm{P} : R \cup \{ip'\}}\ \textsc{Connect}\ (\text{T3-2})$$

$$\frac{}{ip : \mathrm{P} : R \xrightarrow{\mathbf{disconnect}(ip', ip)} ip : \mathrm{P} : R \setminus \{ip'\}}\ \textsc{Disconnect}\ (\text{T3-2})$$

$$\frac{ip \notin \{ip', ip''\}}{ip : \mathrm{P} : R \xrightarrow{\mathbf{connect}(ip', ip'')} ip : \mathrm{P} : R}\ \textsc{Connect}\ (\text{T3-3})$$

$$\frac{ip \notin \{ip', ip''\}}{ip : \mathrm{P} : R \xrightarrow{\mathbf{disconnect}(ip', ip'')} ip : \mathrm{P} : R}\ \textsc{Disconnect}\ (\text{T3-3})$$

$$H \subseteq R \wedge K \cap R = \emptyset\ \frac{\mathrm{M} \xrightarrow{R:*\mathbf{cast}(m)} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{H \neg K:\mathbf{arrive}(m)} \mathrm{N}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{R:*\mathbf{cast}(m)} \mathrm{M}' \parallel \mathrm{N}'}\ \textsc{Cast}\ (\text{T4-1})$$

$$H \subseteq R \wedge K \cap R = \emptyset\ \frac{\mathrm{M} \xrightarrow{H \neg K:\mathbf{arrive}(m)} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{R:*\mathbf{cast}(m)} \mathrm{N}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{R:*\mathbf{cast}(m)} \mathrm{M}' \parallel \mathrm{N}'}\ \textsc{Cast}\ (\text{T4-2})$$

$$\frac{\mathrm{M} \xrightarrow{H \neg K:\mathbf{arrive}(m)} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{H' \neg K':\mathbf{arrive}(m)} \mathrm{N}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{(H \cup H') \neg (K \cup K'):\mathbf{arrive}(m)} \mathrm{M}' \parallel \mathrm{N}'}\ \textsc{Cast}\ (\text{T4-3})$$

$$\frac{\mathrm{M} \xrightarrow{R:*\mathbf{cast}(m)} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\tau} [\ \mathrm{M}'\ ]}\ \textsc{Cast}\ (\text{T4-4})$$

$$\frac{\mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}' \parallel \mathrm{N}}\ \textsc{Deliver}\ (\text{T4-1})$$

$$\frac{\mathrm{M} \xrightarrow{\tau} \mathrm{M}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{\tau} \mathrm{M}' \parallel \mathrm{N}}\ \textsc{Internal}\ (\text{T4-1})$$

$$\frac{\mathrm{N} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{N}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M} \parallel \mathrm{N}'}\ \textsc{Deliver}\ (\text{T4-2})$$

$$\frac{\mathrm{N} \xrightarrow{\tau} \mathrm{N}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{\tau} \mathrm{M} \parallel \mathrm{N}'}\ \textsc{Internal}\ (\text{T4-2})$$

$$\frac{\mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{ip:\mathbf{deliver}(d)} [\ \mathrm{M}'\ ]}\ \textsc{Deliver}\ (\text{T4-3})$$

$$\frac{\mathrm{M} \xrightarrow{\tau} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\tau} [\ \mathrm{M}'\ ]}\ \textsc{Internal}\ (\text{T4-3})$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{connect}(ip, ip')} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{\mathbf{connect}(ip, ip')} \mathrm{N}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{\mathbf{connect}(ip, ip')} \mathrm{M}' \parallel \mathrm{N}'}\ \textsc{Connect}\ (\text{T4-1})$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{disconnect}(ip, ip')} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{\mathbf{disconnect}(ip, ip')} \mathrm{N}'}{\mathrm{M} \parallel \mathrm{N} \xrightarrow{\mathbf{disconnect}(ip, ip')} \mathrm{M}' \parallel \mathrm{N}'}\ \textsc{Disconnect}\ (\text{T4-1})$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{connect}(ip, ip')} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\mathbf{connect}(ip, ip')} [\ \mathrm{M}'\ ]}\ \textsc{Connect}\ (\text{T4-2})$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{disconnect}(ip, ip')} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\mathbf{disconnect}(ip, ip')} [\ \mathrm{M}'\ ]}\ \textsc{Disconnect}\ (\text{T4-2})$$

$$\frac{\mathrm{M} \xrightarrow{\{ip\} \neg K:\mathbf{arrive}(\mathtt{newpkt}(d, dip))} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{ip:\mathbf{newpkt}(d, dip)} [\ \mathrm{M}'\ ]}\ \textsc{Newpkt}\ (\text{T4})$$

Table 2.2: *AWN inference rules (2/2)*

1.4 Network level

A partial network is constructed through parallel composition of nodes (via the || operator):

$$\mathrm{M} ::= \mathit{ip} : \mathrm{PP} : R \quad | \quad \mathrm{M} \lll \mathrm{M}$$

Parallellized nodes can *exchange messages* according to rules Cast (T4-1) to Cast (T4-4) in Table 2.2. These exchanges occur through synchronization of R : **starcast**(m) and $H\neg K$: **arrive**(m) actions (where $\neg$ is simply a syntactic separator of the sets H and K). Of these two actions, $H\neg K$: **arrive**(m) is where a message m arrives at the nodes in the set H and where m is ignored by the nodes in the set K (because they are out of range). It is necessary for synchronization that $H \subseteq R$ and that $K \cap R = \emptyset$ because R in R : **starcast**(m) is the set of addresses of the intended recipients within range.
Encapsulation ($[\_]$) allows a partial network M to be converted into a complete network $[M]$. Encapsulation also restricts the set of actions exposed by a network to τ, **connect**$(\mathit{ip}, \mathit{ip'})$ and **disconnect**$(\mathit{ip}, \mathit{ip'})$, ip : **newpkt**(data, d), and ip : **deliver**(data).
These actions have the following meanings. As usual in process algebra, τ is an action that is hidden from the environment of the network. Actions **connect**$(\mathit{ip}, \mathit{ip'})$ and **disconnect**$(\mathit{ip}, \mathit{ip'})$ represent events within the network where a node with address *ip'* respectively enters or leaves the range of a node with address *ip* (note that all nodes must agree on this topology change). Action ip : **newpkt**(data, d) is the event where a data object d to be delivered at the node with address d is injected into the network (wrapped into a message `newpkt`(data, d)) at the node with address *ip*, whereas action ip : **deliver**(data) is the actual delivery of a data object at the node with address *ip*. The accompanying inference rules are Deliver (T4-1) to Newpkt (T4) in Table 2.2.

1.5 AWN examples

Listings 2.1 to 2.4 contain simple examples to illustrate how AWN might be used. The example of Listing 2.1 defines a radio tower (`RadioTower`, line 1) that broadcasts messages to radios (`Radio`, line 4) within range. Each message contains a value 1 higher than the value of the preceding message, and radios store the most recently received message. A specific network related to this example might be defined as in lines 6 through 9: the radio tower has address 1, there are 3 radios with addresses 2 to 4, and only radios with addresses 2 and 4 are within range of the radio tower.

```
1  RadioTower(msgVal: Integer) =
2    broadcast(new Message(msgVal)) . RadioTower(msgVal + 1);
3
4  Radio(lastMsg: Message) = receive(msg) . Radio(msg);
5
6  RadioNetwork = [ 1 : RadioTower(1) : { 2, 4 } ||
7                   2 : Radio(null) : ∅ ||
8                   3 : Radio(null) : ∅ ||
9                   4 : Radio(null) : ∅ ];
```

Listing 2.1: *AWN radio tower example.*

The second example introduces a 'chat client' (`ChatClient`, line 1) that listens for messages similar to the `Radio` process. However, the server (`ChatServer`, line 6) only forwards messages that it receives from clients within range. These messages are non-deterministically sent by clients to a node within range that has address 1, which is assumed to be the chat server. Clients always send the same message.

```
1  ChatClient(lastMsg: Message, sentMsg: Boolean) =
2      receive(msg) . ChatClient(msg, sentMsg)
3    + unicast(1, Message("Hello everyone!")) .
4        ChatClient(lastMsg, true) ▶ ChatClient(lastMsg, false);
5
6  ChatServer() = receive(msg) . broadcast(msg) . ChatServer();
```

Listing 2.2: *AWN 'chat client' example.*

```
1  Flood(msgSent: Boolean, store: IP ↦ Text) =
2      receive(msg) . [msg = Message(ip', text)] (
3        [ip' ↦ text ∈ store] . Flood(msgSent, store)
4      + [ip' ↦ text ∉ store] .
5          ⟦ store := store[ip' ↦ text] ⟧
6            broadcast(msg) . Flood(msgSent, store))
7    + [¬ msgSent] broadcast(new Message("Hello everyone!")) .
8        Flood(true, store);
```

Listing 2.3: *AWN flooding example.*

Listing 2.3 defines a simple flooding protocol. Each node of a network runs the same process (`Flood`), line 1). This process injects its message into the network at a non-deterministic time (similar to `ChatClient` from the previous example, but now nodes send their message a maximum of one time). The process also listens for incoming messages: for each node address, it stores the contents of the first message received from that node. When analyzed, the example of Listing 2.3 reveals a problem with the protocol: the network can deadlock when some nodes A and B have just received a message (at the end of line 2) that is not in their `store` (line 4) and when either A or B needs to forward its message to node B in order for it to complete its **broadcast** action. The other node is essentially 'not ready to receive', also described as not being *input-enabled*.

```
1  Queue(queue: Sequence(Message)) =
2      receive(msg) . Queue(queue→append(msg))
3    + [queue→size() > 0] (
4        receive(msg) . Queue(queue→append(msg))
5      + send(queue→at(0)) . Queue(queue→remove(0))
6    );
7
8  FloodWithQueue = Flood(false, ∅) << Queue(∅);
9
10 FloodNetwork = [ 1 : FloodWithQueue() : { 2, 3, 4 } ||
11                  2 : FloodWithQueue() : { 1, 3 } ||
12                  3 : FloodWithQueue() : { 1, 2 } ||
13                  4 : FloodWithQueue() : { 1 } ];
```

Listing 2.4: *AWN queued flooding example.*

The example of Listing 2.4 uses the typical approach to make a protocol specification input-enabled: in addition to the original `Flood` process, each node is also running the `Queue` process (line 8). Because the `Queue` process is input-enabled (note the seemingly superfluous **receive** on line 4, which makes it possible to receive a message in the process state just after the guard of line 3 has been evaluated), the `Flood` process can take more time to handle incoming messages without causing deadlocks.

2 mCRL2 semantics

For the translation from AWN to mCRL2, the features of mCRL2 related to time will not be used. This section will therefore give the *untimed* fragment of the semantics of mCRL2 (a similar approach as the one used in [22]). Note that only an overview is provided here; refer to the mCRL2 book [**TODO**] for the full semantics.

2.1 Grammar

Similar to sequential processes in AWN, mCRL2 processes have a signature $X(\mathtt{var}_1 : D_1, \cdots, \mathtt{var}_n : D_n)$ where X is a name and $\mathtt{var}_i$ are process parameters of data type D_i. All mCRL2 processes are contained within the set *PD*. The behavior of mCRL2 processes is specified by expressions conforming to the following grammar:

$$\begin{aligned} \mathrm{p} \;::=\;& \delta \mid \omega \mid \mathrm{p.p} \mid \mathrm{p}+\mathrm{p} \mid c \to \mathrm{p} \mid c \to \mathrm{p} \diamond \mathrm{p} \mid \textstyle\sum_{\mathtt{x:D}} \mathrm{p} \\ & \mid \mathrm{p} \parallel \mathrm{p} \mid \Gamma_C(\mathrm{p}) \mid \nabla_V(\mathrm{p}) \mid \partial_B(\mathrm{p}) \mid \rho_R(\mathrm{p}) \mid \tau_I(\mathrm{p}) \mid \mathrm{X}(\mathtt{d}_1, \cdots, \mathtt{d}_n) \\ \omega \;::=\;& \tau \mid \mathbf{a}(\mathtt{d}_1, \cdots, \mathtt{d}_n) \mid \omega|\omega \end{aligned}$$

In this grammar, δ represents a deadlocked process, meaning that it cannot make any transitions. To do an action ω, one only has to write it; however, mCRL2 actually uses *multi-actions*, which are actions that potentially consist of multiple action labels **a** that can each carry their own data. The action τ is the empty multi-action with the property that $\omega|\tau = \tau|\omega = \omega$.
Processes can be chained, or choices can be made between them, either by checking a condition c or non-deterministically – the expression $\sum_{\mathtt{x:D}} \mathrm{p}$, in particular, is used to express a non-deterministic choice between summands for every possible value of x, a variable of data type D. Expressions p and q can be put in parallel by writing $\mathrm{p} \parallel \mathrm{q}$. Finally Γ_C, ∇_V, ∂_B, ρ_R, and τ_I are operators that can influence the behavior of a process in various ways – their descriptions can be found in 2.3.

2.2 Inference rules

Table 2.3 lists rules in Plotkin style [19] for the structural operational semantics of mCRL2 process expressions. In these rules, several notations are used that require short explanations:

- The ✓-predicate is used to indicate the mCRL2 termination state – a process 'goes here' when it has no more actions to do;
- The $[\![exp]\!]$-brackets denotes the semantic value of *exp*;
- The syntax $\mathtt{d} : \mathrm{D}$ means that variable d is of type D (although 'types' are called 'sorts' in mCRL2);
- M_{D} refers to the set of all possible semantic values of type D; Furthermore, if $e \in M_{\mathrm{D}}$ then there exists $t_{\mathrm{U}(e)}$, a syntactic symbol for e such that $[\![t_{\mathrm{U}(e)}]\!] = e$;
- $\underline{\omega}$ denotes a multi-action from which all data has been removed;
- The expression $\omega \underline{\in} V$ results to true if and only if an equivalent of multi-action ω is in the set of multi-actions V (for example, $\mathbf{a}|\mathbf{a}|\mathbf{b} \underline{\in} \{\mathbf{a}|\mathbf{b}|\mathbf{a}\} = \mathit{true}$ because two multi-actions that differ only in the ordering of their action labels are equivalent);
- $\omega_{\{\}}$ is the set of all single actions that multi-action ω contains.

2.3 Special operators

The ∇_V, ∂_B, Γ_C, ρ_R, and τ_I are special operators that affect the actions that a process can perform. The following subsections describe these operators in greater detail.

Allow operator

The allow operator ∇_V only allows τ and actions and multi-actions that are in the set of multi-actions V from occurring in the operand. Its behavior follows Allow 1 and Allow 2 in Table 2.3.

Blocking operator

The blocking operator ∂_B (see Block 1 and Block 2) prevents actions that are in B from occurring in the operand (with the exception of τ). Any (multi-)action sharing an action with B is blocked – the blocked actions are not 'subtracted' from their multi-actions!

$$\frac{}{\alpha \xrightarrow{[\![\alpha]\!]} \checkmark}\ \textsc{Axiom}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\mathrm{p}+\mathrm{q} \xrightarrow{\omega} \checkmark}\ \textsc{Choice 1} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\mathrm{p}.\mathrm{q} \xrightarrow{\omega} \mathrm{q}}\ \textsc{Seq 1} \qquad [\![c]\!] = \mathit{true}\ \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{c \to \mathrm{p} \diamond \mathrm{q} \xrightarrow{\omega} \checkmark}\ \textsc{Guard 3}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\mathrm{p}+\mathrm{q} \xrightarrow{\omega} \mathrm{p}'}\ \textsc{Choice 2} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\mathrm{p}.\mathrm{q} \xrightarrow{\omega} \mathrm{p}'.\mathrm{q}}\ \textsc{Seq 2} \qquad [\![c]\!] = \mathit{true}\ \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{c \to \mathrm{p} \diamond \mathrm{q} \xrightarrow{\omega} \mathrm{p}'}\ \textsc{Guard 4}$$

$$\frac{\mathrm{q} \xrightarrow{\omega} \checkmark}{\mathrm{p}+\mathrm{q} \xrightarrow{\omega} \checkmark}\ \textsc{Choice 3} \qquad [\![c]\!] = \mathit{true}\ \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{c \to \mathrm{p} \xrightarrow{\omega} \checkmark}\ \textsc{Guard 1} \qquad [\![c]\!] = \mathit{false}\ \frac{\mathrm{q} \xrightarrow{\omega} \checkmark}{c \to \mathrm{p} \diamond \mathrm{q} \xrightarrow{\omega} \checkmark}\ \textsc{Guard 5}$$

$$\frac{\mathrm{q} \xrightarrow{\omega} \mathrm{q}'}{\mathrm{p}+\mathrm{q} \xrightarrow{\omega} \mathrm{q}'}\ \textsc{Choice 4} \qquad [\![c]\!] = \mathit{true}\ \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{c \to \mathrm{p} \xrightarrow{\omega} \mathrm{p}'}\ \textsc{Guard 2} \qquad [\![c]\!] = \mathit{false}\ \frac{\mathrm{q} \xrightarrow{\omega} \mathrm{q}'}{c \to \mathrm{p} \diamond \mathrm{q} \xrightarrow{\omega} \mathrm{q}'}\ \textsc{Guard 6}$$

$$\mathrm{P}(\mathtt{d}_1 : \mathrm{D}_1, \cdots, \mathtt{d}_n : \mathrm{D}_n) \stackrel{\text{def}}{=} \mathrm{q}\ \frac{\mathrm{q}[\mathtt{d}_1 := t_1, \cdots, \mathtt{d}_n := t_n] \xrightarrow{\omega} \checkmark}{\mathrm{P}(t_1, \cdots, t_n) \xrightarrow{\omega} \checkmark}\ \textsc{Recursion 1}$$

$$\mathrm{P}(\mathtt{d}_1 : \mathrm{D}_1, \cdots, \mathtt{d}_n : \mathrm{D}_n) \stackrel{\text{def}}{=} \mathrm{q}\ \frac{\mathrm{q}[\mathtt{d}_1 := t_1, \cdots, \mathtt{d}_n := t_n] \xrightarrow{\omega} \mathrm{q}'}{\mathrm{P}(t_1, \cdots, t_n) \xrightarrow{\omega} \mathrm{q}'}\ \textsc{Recursion 2}$$

$$e \in M_{\mathrm{D}}\ \frac{\mathrm{p}[\mathtt{d} := t_e] \xrightarrow{\omega} \checkmark}{\sum_{\mathtt{d}:\mathrm{D}} \mathrm{p} \xrightarrow{\omega} \checkmark}\ \textsc{Sum 1} \qquad \underline{\omega} \in V \cup \{\tau\}\ \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\nabla_V(\mathrm{p}) \xrightarrow{\omega} \checkmark}\ \textsc{Allow 1}$$

$$e \in M_{\mathrm{D}}\ \frac{\mathrm{p}[\mathtt{d} := t_e] \xrightarrow{\omega} \mathrm{p}'}{\sum_{\mathtt{d}:\mathrm{D}} \mathrm{p} \xrightarrow{\omega} \mathrm{p}'}\ \textsc{Sum 2} \qquad \underline{\omega} \in V \cup \{\tau\}\ \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\nabla_V(\mathrm{p}) \xrightarrow{\omega} \nabla_V(\mathrm{p}')}\ \textsc{Allow 2}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega} \mathrm{q}}\ \textsc{Par 1} \qquad \omega_{\{\}} \cap B = \emptyset\ \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\partial_B(\mathrm{p}) \xrightarrow{\omega} \checkmark}\ \textsc{Block 1}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega} \mathrm{p}' \mid\mid \mathrm{q}}\ \textsc{Par 2} \qquad \omega_{\{\}} \cap B = \emptyset\ \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\partial_B(\mathrm{p}) \xrightarrow{\omega} \partial_B(\mathrm{p}')}\ \textsc{Block 2}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}' \qquad \mathrm{q} \xrightarrow{\overline{\omega}} \mathrm{q}'}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega|\overline{\omega}} \mathrm{p}' \mid\mid \mathrm{q}'}\ \textsc{Par 3} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\rho_R(\mathrm{p}) \xrightarrow{R \bullet \omega} \checkmark}\ \textsc{Rename 1}$$

$$\frac{\mathrm{q} \xrightarrow{\omega} \checkmark}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega} \mathrm{p}}\ \textsc{Par 4} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\rho_R(\mathrm{p}) \xrightarrow{R \bullet \omega} \rho_R(\mathrm{p}')}\ \textsc{Rename 2}$$

$$\frac{\mathrm{q} \xrightarrow{\omega} \mathrm{q}'}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega} \mathrm{p} \mid\mid \mathrm{q}'}\ \textsc{Par 5} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\Gamma_C(\mathrm{p}) \xrightarrow{\gamma_C(\omega)} \checkmark}\ \textsc{Comm 1}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \checkmark \qquad \mathrm{q} \xrightarrow{\overline{\omega}} \checkmark}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega|\overline{\omega}} \checkmark}\ \textsc{Par 6} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\Gamma_C(\mathrm{p}) \xrightarrow{\gamma_C(\omega)} \Gamma_C(\mathrm{p}')}\ \textsc{Comm 2}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}' \qquad \mathrm{q} \xrightarrow{\overline{\omega}} \checkmark}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega|\overline{\omega}} \mathrm{p}'}\ \textsc{Par 7} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \checkmark}{\tau_I(\mathrm{p}) \xrightarrow{\theta_I(\omega)} \checkmark}\ \textsc{Hide 1}$$

$$\frac{\mathrm{p} \xrightarrow{\omega} \checkmark \qquad \mathrm{q} \xrightarrow{\overline{\omega}} \mathrm{q}'}{\mathrm{p} \mid\mid \mathrm{q} \xrightarrow{\omega|\overline{\omega}} \mathrm{q}'}\ \textsc{Par 8} \qquad \frac{\mathrm{p} \xrightarrow{\omega} \mathrm{p}'}{\tau_I(\mathrm{p}) \xrightarrow{\theta_I(\omega)} \tau_I(\mathrm{p}')}\ \textsc{Hide 2}$$

Table 2.3: *mCRL2 inference rules*

Communication operator

The communication operator Γ_C (see Comm 1 and Comm 2) modifies multi-actions that contain combinations of action labels by renaming those combinations to a specific action label. To this end, it requires a set of pairs $(\omega \rightarrow \mathbf{a})$ where ω is a multi-action without data and **a** an action label. This set is passed to an auxiliary function γ, which is defined as follows:

$$\begin{aligned}
\gamma_\emptyset(\alpha) &= \alpha \\
\gamma_{C_1 \cup C_2}(\alpha) &= \gamma_{C_1}(\gamma_{C_2}(\alpha)) \\
\gamma_{\{a_1|\ldots|a_n \rightarrow b\}}(\alpha) &= \begin{cases} b(\mathbf{d})|\gamma_C(\alpha \setminus (a_1(\overline{d})|\ldots|a_n(\overline{d}))) \text{ if } (a_1(\overline{d})|\ldots|a_n(\overline{d})) \sqsubseteq \alpha \text{ for some vector } \overline{d} \\ \alpha \text{ otherwise} \end{cases}
\end{aligned}$$

where the operators $\setminus$ and $\sqsubseteq$ – although likely intuitively understood – are defined as

$$\begin{aligned}
\tau \setminus \alpha &= \tau \\
\alpha \setminus \tau &= \alpha \\
\alpha \setminus (\beta|\gamma) &= (\alpha \setminus \beta) \setminus \gamma \\
(\alpha|a(d_1, \cdots, d_n)) \setminus a(d_1, \cdots, d_n) &= \alpha \\
(\alpha|a(d_1, \cdots, d_n)) \setminus b(e_1, \cdots, e_n) &= (\alpha \setminus b(e_1, \cdots, e_n))|a(d_1, \cdots, d_n) \text{ if } a \neq b \vee d_1 \neq e_1 \vee \cdots \vee d_n \neq e_n
\end{aligned}$$

and

$$\begin{aligned}
\tau \sqsubseteq \alpha &= \mathit{true} \\
a(d_1, \cdots, d_n) \sqsubseteq \tau &= \mathit{false} \\
\alpha|a(d_1, \cdots, d_n) \sqsubseteq \beta|a(d_1, \cdots, d_n) &= \alpha \sqsubseteq \beta \\
\alpha|a(d_1, \cdots, d_n) \sqsubseteq \beta|b(e_1, \cdots, e_n) &= (\alpha \setminus b(e_1, \cdots, e_n))|a(d_1, \cdots, d_n) \sqsubseteq \beta \text{ if } a \neq b \vee d_1 \neq e_1 \vee \cdots \vee d_n \neq e_n
\end{aligned}$$

respectively.

Renaming operator

The renaming operator ρ_R renames all actions in the operand with the name a to b for all $a \rightarrow b \in R$ (where $a \neq \tau$, $b \neq \tau$). To this end, the following auxiliary function is used in Table 2.3:

$$\begin{aligned}
R \bullet \tau &= \tau \\
R \bullet a(d_1, \cdots, d_n) &= b(d_1, \cdots, d_n) \text{ if } a \rightarrow b \in R \text{ for some } b \\
R \bullet a(d_1, \cdots, d_n) &= a(d_1, \cdots, d_n) \text{ if } a \rightarrow b \notin R \text{ for all } b \\
R \bullet (\alpha|\beta) &= (R \bullet \alpha)|(R \bullet \beta)
\end{aligned}$$

Hiding operator

The hiding operator τ_I replaces all actions in the operand with the name a by τ for all $a \in I$. Due to the property that $\omega|\tau = \tau|\omega = \omega$, the a is simply removed instead if a is part of a multi-action.
The Block 1 and Block 2 inference rules in Table 2.3 describe the behavior of the hiding operator by using a function θ. This function is defined as follows:

$$\begin{aligned}
\theta_I(\tau) &= \tau \\
\theta_I(a(d_1, \cdots, d_n)) &= \tau \text{ if } a \in I \\
\theta_I(a(d_1, \cdots, d_n)) &= a(d_1, \cdots, d_n) \text{ if } a \notin I \\
\theta_I(\alpha|\beta) &= \theta_I(\alpha)|\theta_I(\beta)
\end{aligned}$$

2.4 mCRL2 examples

Listings 2.5 to 2.7 give an example of a system specification in mCRL2. Listing 2.5 first specifies some of the required actions in the `act` block (lines 1 to 3). It then defines the two subsystems that constitute the television system as two separate processes: the `Volume` subsystem (lines 6 to 8) allows the volume of the television to be turned up or down within the range 0 to 50, and the `Channel` subsystem (lines 10 to 12) allows the channel (0 to 99) of the television to be changed by twice pressing a `0` to `9` button. The two subsystems can be easily composed using parallel composition (line 14). The `init` section specifies the first state of the entire television system.

```
1  act
2    volume_down, volume_up;
3    press: Integer;
4
5  proc
6    Volume(volume: Integer) =
7        volume_down . Volume(if(volume > 0, volume - 1, 0))
8      + volume_up . Volume(if(volume < 50, volume + 1, 50));
9
10   Channel(channel: Integer) =
11     sum n1:{0..9} . press(n1) .
12       sum n2:{0..9} . press(n2) . Channel(n1 * 10 + n2);
13
14   Television = Channel(1) || Volume(25);
15
16 init
17   Television;
```

Listing 2.5: *Simple mCRL2 television example.*

Listing 2.6 extends the television by adding a mute subsystem. This subsystem allows the user to turn the volume of the television on or off in its entirety by pressing the 'mute' button (line 6). The volume of the television is also turned back on when the user changes the volume of the television (line 7 and 8).

```
1  act
2    mute;
3
4  proc
5    Mute(muted: Boolean) =
6        mute . Mute(!muted)
7      + volume_down . Mute(false)
8      + volume_up . Mute(false);
9
10   Television2 = Channel(1) || Volume(25) || Mute(false);
```

Listing 2.6: *Naive extension of the simple mCRL2 television example.*

Naively, one might be tempted to add the new subsystem to the television by using another parallel composition as in line 10 of Listing 2.6. However, in that case the `volume_up` and `volume_down` actions of the `Volume` and `Mute` subsystems would be independent, and it is therefore possible that the television model changes the volume without unmuting.
Instead, one should make use of the communication operator Γ and the blocking operator ∂, which are discussed in subsections 2.3 and 2.3, respectively. Listing 2.7 shows how this can be accomplished: with the communication operator, the `volume_up` and `volume_down` actions of the `Volume` and `Mute` subsystems are merged (and renamed to `synced_volume_up` and `synced_volume_down`, respectively) and then prevents any independent occurrences of `volume_up` and `volume_down` with the blocking operator.

```
1  act
2    synced_volume_down, synced_volume_up;
3
4  proc
5    Television3 = Channel(1) || ∂{volume_up,volume_down}
6        Γ{volume_up|volume_up→synced_volume_up,volume_down|volume_down→synced_volume_down}
7          (Volume(25) || Mute(false));
```

Listing 2.7: *Intended extension of the simple mCRL2 television example.*

3 Translation function

The project was continued by designing a *translation function* from AWN to mCRL2. The function has been defined by a list of *translation rules* which have been categorized according to one of two purposes, namely for translating AWN's sequential process expressions or for translating higher-level process expressions of AWN. The translation rules for both purposes are discussed in the following sections.

3.1 Translating sequential process expressions

The rules for translating sequential process expressions can be found in Table 2.4.

$$\mathrm{T}_V(\xi, \textbf{broadcast}(ms).\mathrm{p}) = \textstyle\sum_{\mathtt{D}:\mathrm{Set(IP)}} \textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{D} \notin \mathrm{VARS}(\mathrm{T}_\xi(ms)) \cup \mathrm{VARS}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \text{T1}$$

$$\mathrm{T}_V(\xi, \textbf{groupcast}(dests, ms).\mathrm{p}) = \textstyle\sum_{\mathtt{D}:\mathrm{Set(IP)}} \textbf{cast}(\mathrm{T}_\xi(dests), \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{D} \notin \mathrm{VARS}(\mathrm{T}_\xi(dests)) \cup \mathrm{VARS}(\mathrm{T}_\xi(ms)) \cup \mathrm{VARS}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \text{T2}$$

$$\mathrm{T}_V(\xi, \textbf{unicast}(dest, ms).\mathrm{p} \blacktriangleright \mathrm{q}) = \textbf{cast}(\{\mathrm{T}_\xi(dest)\}, \{\mathrm{T}_\xi(dest)\}, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) + \neg\textbf{uni}(\{\mathrm{T}_\xi(dest)\}, \emptyset, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{q}) \quad \text{T3}$$

$$\mathrm{T}_V(\xi, \textbf{send}(\mathrm{T}_\xi(ms)).\mathrm{p}) = \textbf{send}(\emptyset, \emptyset, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \quad \text{T4}$$

$$\mathrm{T}_V(\xi, \textbf{deliver}(data).\mathrm{p}) = \textstyle\sum_{\mathtt{ip}:\mathrm{IP}} \textbf{del}(\mathtt{ip}, \mathrm{T}_\xi(data)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{ip} \notin \mathrm{VARS}(\mathrm{T}_\xi(data)) \cup \mathrm{VARS}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \text{T5}$$

$$\mathrm{T}_V(\xi, \textbf{receive}(\mathtt{msg}).\mathrm{p}) = \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathrm{T}(\mathtt{msg}):\mathrm{MSG}} \textbf{receive}(\mathtt{D}, \mathtt{D'}, \mathrm{T}(\mathtt{msg})).\mathrm{T}_{V \cup \{\mathtt{msg}\}}(\xi, \mathrm{p}) \text{ where } \mathtt{D}, \mathtt{D'} \notin \mathrm{VARS}(\mathrm{T}_{V \cup \{\mathtt{msg}\}}(\xi, \mathrm{p})) \quad \text{T6}$$

$$\mathrm{T}_V(\xi, [\![\mathtt{var} := exp]\!]\ \mathrm{p}) = \textstyle\sum_{\mathtt{tmp}:\mathrm{sort}(\mathrm{T}(\mathtt{var}))} (\mathtt{tmp} = \mathrm{T}_\xi(exp)) \rightarrow \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))} (\mathrm{T}(\mathtt{var}) = \mathtt{tmp}) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{V \cup \{\mathtt{var}\}}(\xi, \mathrm{p}) \quad \text{T7}$$
$$\text{where } \mathrm{T}(\mathtt{tmp}) \notin \mathrm{VARS}(\mathrm{T}_\xi(exp)) \cup \mathrm{VARS}(\mathrm{T}_{V \cup \{\mathtt{var}\}}(\xi, \mathrm{p}))$$

$$\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) = \mathrm{X}(\mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}_\xi(exp_n)) \quad \text{T8}$$
$$\text{where } \mathrm{T}(\mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}) = \mathrm{X}(\mathrm{T}(\mathtt{var}_1) : \mathrm{sort}(\mathrm{T}_\xi(exp_1)), \cdots, \mathrm{T}(\mathtt{var}_n) : \mathrm{sort}(\mathrm{T}_\xi(exp_n))) \stackrel{\text{def}}{=} \mathrm{T}_{\{\mathtt{var}_1, \cdots, \mathtt{var}_n\}}(\emptyset, \mathrm{p})$$

$$\mathrm{T}_V(\xi, \mathrm{p} + \mathrm{q}) = \mathrm{T}_V(\xi, \mathrm{p}) + \mathrm{T}_V(\xi, \mathrm{q}) \quad \text{T9}$$

$$\mathrm{T}_V(\xi, [\phi]\mathrm{p}) = \textstyle\sum_{\mathrm{T}(\mathrm{Fv}(\phi) \setminus V)} \mathrm{T}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{V \cup \mathrm{Fv}(\phi)}(\xi, \mathrm{p}) \quad \text{T10}$$

$$\mathrm{T}(\mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}) = \mathrm{X}(\mathrm{T}(\mathtt{var}_1) : \mathrm{sort}(\mathrm{T}(\mathtt{var}_1)), \cdots, \mathrm{T}(\mathtt{var}_n) : \mathrm{sort}(\mathrm{T}(\mathtt{var}_n))) \stackrel{\text{def}}{=} \mathrm{T}_{\{\mathtt{var}_1, \cdots, \mathtt{var}_n\}}(\emptyset, \mathrm{p}) \quad \text{T11}$$

Table 2.4: *Rules for translating AWN's sequential process expressions*

The translation rules in Table 2.4 use $\mathrm{T}_V(\xi, \mathrm{p})$ as the signature of the translation function, where p is the sequential process expression to be translated, ξ is a *valuation* (a mapping from variables to semantic values), and V is a set of AWN variables (those that are assigned up to this point, to be precise). Note that ξ is read-only: it is carried around exclusively for the benefit of the $\mathrm{T}_\xi(exp)$ function, which translates data expressions. The definition of $\mathrm{T}_\xi(exp)$ is relatively complex:

$$\mathrm{T}_\xi(exp) \stackrel{\text{def}}{=} \mathcal{F}\left(\mathrm{T}(exp)\left[\mathrm{T}(\mathtt{x}) := t_{\mathrm{U}(y)} \;\middle|\; \mathtt{x} := y \in \xi\right]\right)$$

where

$$\mathcal{F}(e) \stackrel{\text{def}}{=} \begin{cases} t_{[\![e]\!]} & \text{if } \mathrm{Fv}(e) = \emptyset \\ e & \text{otherwise;} \end{cases}$$

where $\mathrm{U}(y)$ translates a semantic value of AWN called y to the corresponding semantics value of mCRL2; where the Axiom of Choice is applied to the selection of t_Y (given a particular semantic value Y, the same syntactic representation t_Y will always be selected); and where the function $\mathrm{T}(exp)$ translates a syntactic AWN data expression *exp* to a syntactic mCRL2 data expression. In order to abstract from the translation of data expressions as much as possible, $\mathrm{T}(exp)$ is not further defined but simply assumed to exist with the following properties:

(i) $\mathrm{T}(exp)$ is a total function;
(ii) $\mathrm{T}(exp)$ behaves in such a way that $\forall exp, \xi\ .\ \xi(exp)$ is defined $\Rightarrow [\![\mathrm{T}_\xi(exp)]\!] = \mathrm{U}(\xi(exp))$;
(iii) $\forall exp\ .\ \mathrm{Fv}(\mathrm{T}(exp)) = \{\mathrm{T}(\mathtt{v}) \mid \mathtt{v} \in \mathrm{Fv}(exp)\}$; and
(iv) $exp = \mathtt{v} \Leftrightarrow \mathrm{T}(exp) = \mathtt{v'}$ where v is an AWN variable and v' is an mCRL2 variable.

The following subsections will elaborate on each of the translation rules in Table 2.4.

Rule T1: Broadcast

$$\mathrm{T}_V(\xi, \mathbf{broadcast}(ms).\mathrm{p}) = \sum_{\mathtt{D:Set(IP)}} \mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \tau_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{D} \notin \textsc{vars}(\tau_\xi(ms)) \cup \textsc{vars}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \text{T1}$$

This rule defines how a **broadcast** action is translated to mCRL2. The resulting **cast** action has three parameters and is then immediately followed by the translation of p.
The first parameter of the **cast** action is the set of addresses of nodes that are the intended recipients of the message *ms*. Since a **broadcast** action always attempts to reach all nodes with range, the value of this parameter is the universe of all node addresses ($\mathcal{U}_{\mathrm{IP}}$).
The second parameter is the set of actual recipients. The sequential process does not have direct access to which nodes are within range, however, and the value of this parameter is therefore unknown. This has been solved by adding $\sum_{\mathtt{D:Set(IP)}}$, the resulting mCRL2 expression produces a **cast** action for *all* possible values – superfluous actions will be eliminated at a later stage. The side condition of T1 ensures that D is fresh.
The third parameter is the message that is sent translated to an mCRL2 expression by means of the $\tau_\xi(exp)$ function.

Rule T2: Groupcast

$$\mathrm{T}_V(\xi, \mathbf{groupcast}(dests, ms).\mathrm{p}) = \sum_{\mathtt{D:Set(IP)}} \mathbf{cast}(\tau_\xi(dests), \mathtt{D}, \tau_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{D} \notin \textsc{vars}(\tau_\xi(dests)) \cup \textsc{vars}(\tau_\xi(ms)) \cup \textsc{vars}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \text{T2}$$

This rule defines how a **groupcast** action is translated to mCRL2. Just like for the translation of **broadcast**, the resulting **cast** action has three parameters and is then immediately followed by the translation of p. The second and third parameter are also translated the same as for **broadcast** (with the side condition of T2 ensuring that D is fresh), but the first parameter is not: rather than $\mathcal{U}_{\mathrm{IP}}$ the set of destinations specified by the user is inserted. The first parameter must contain the node addresses of the *intended* recipients, after all.

Rule T3: Unicast

$$\mathrm{T}_V(\xi, \mathbf{unicast}(dest, ms).\mathrm{p} \blacktriangleright \mathrm{q}) = \mathbf{cast}(\{\tau_\xi(dest)\}, \{\tau_\xi(dest)\}, \tau_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) + \neg\mathbf{uni}(\{\tau_\xi(dest)\}, \emptyset, \tau_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{q}) \quad \text{T3}$$

The translation of the **unicast** action produces two possible outcomes: if the destination node is within range, a **cast** action can occur, sending message *ms* to the destination specified by the user and continuing by executing p; and it the destination node is *not* within range, a $\neg$**uni** action can occur, followed by the execution of q.

Rule T4: Send

$$\mathrm{T}_V(\xi, \mathbf{send}(\tau_\xi(ms)).\mathrm{p}) = \mathbf{send}(\emptyset, \emptyset, \tau_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \quad \text{T4}$$

The translation rule for the **send** action is the simplest of them all: it produces a single action by the same name and with three parameters. The first and second parameter of this action are simply initialized with an empty set of node addresses (these parameters are strictly necessary because the signature of **send** must match the signature of the **receive** action) and the third parameter carries the message *ms* that is sent.

Rule T5: Deliver

$$\mathrm{T}_V(\xi, \mathbf{deliver}(data).\mathrm{p}) = \sum_{\mathtt{ip:IP}} \mathbf{del}(\mathtt{ip}, \tau_\xi(data)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{ip} \notin \textsc{vars}(\tau_\xi(data)) \cup \textsc{vars}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \text{T5}$$

The **deliver** action allows the contents of messages (data) to 'leave' a network. Such an event involves two items of information in AWN: the data that is leaving the network, and the node where the data is leaving. Accordingly, the rule above produces a **del** action with these two items as its parameters.
However, a sequential process is unaware of the node on which it is running. The chosen solution for this problem is that a **del** action is produced for every possible node address ip – superfluous actions will be eliminated at a later stage. The side condition of the rule ensures that the variable ip is fresh.

Rule T6: Receive

$$\mathrm{T}_V(\xi, \textbf{receive}(\texttt{msg}).\mathrm{p}) = \sum_{\texttt{D},\texttt{D'}:\mathrm{Set(IP)},\mathrm{T}(\texttt{msg}):\mathrm{MSG}} \textbf{receive}(\texttt{D}, \texttt{D'}, \mathrm{T}(\texttt{msg})).\mathrm{T}_{V \cup \{\texttt{msg}\}}(\xi, \mathrm{p}) \text{ where } \texttt{D}, \texttt{D'} \notin \textsc{vars}(\mathrm{T}_{V \cup \{\texttt{msg}\}}(\xi, \mathrm{p})) \quad \text{T6}$$

When receiving a message, a node at the level of a sequential process expression is unaware of three things: the intended recipients of the message, `D`; the actual recipients of the message, `D'`; and the contents of the message. A sequential process therefore considers all possible **receive** actions that could occur, and relies on the composition with other parts of the specification to eliminate the superfluous ones.
The process p is translated with the additional information that `msg` is now assigned, which is of course because it contains the contents of the received message.

Rule T7: Assignment

$$\mathrm{T}_V(\xi, [\![\texttt{var} := \mathit{exp}]\!]\ \mathrm{p}) = \sum_{\texttt{tmp}:\mathrm{sort}(\mathrm{T}(\texttt{var}))} (\texttt{tmp} = \mathrm{T}_\xi(\mathit{exp})) \rightarrow \sum_{\mathrm{T}(\texttt{var}):\mathrm{sort}(\mathrm{T}(\texttt{var}))} (\mathrm{T}(\texttt{var}) = \texttt{tmp}) \rightarrow \mathbf{t}(\emptyset, \emptyset, \texttt{msg}_{\mathit{dummy}}).\mathrm{T}_{V \cup \{\texttt{var}\}}(\xi, \mathrm{p}) \quad \text{T7}$$
$$\text{where } \mathrm{T}(\texttt{tmp}) \notin \textsc{vars}(\mathrm{T}_\xi(\mathit{exp})) \cup \textsc{vars}(\mathrm{T}_{V \cup \{\texttt{var}\}}(\xi, \mathrm{p}))$$

mCRL2 does not provide its own syntax for assignments other than when parameter values are assigned when a new process is instantiated. A more improvised solution is used here, namely one where a $\sum$ operator and a guard ensure that a specific variable matches the assigned value (see the innermost/rightmost $\sum$ operator). The assigned value is fixed beforehand by a similar construction (see the outermost/leftmost $\sum$ operator) in order to allow the target variable to be used in the computation of the assigned value. The variable in which the assigned value is stored, `tmp`, is fresh as a result of the side condition of the rule.
One might wonder about the three placeholder parameters of the **t**, or indeed about the reason why **t** was used rather than τ in the first place. The answer to the first riddle is that actions that *do* carry three useful parameters will be renamed to **t** at a later stage, and one of the requirements of mCRL2 for renaming an action a to b is that the action signatures of a and b are identical. The answer to the second riddle is that the concurrent behavior of the τ action in mCRL2 does not match the behavior of the τ action in AWN, and that it is necessary to treat τ as a 'regular' action until the network level has been reached.

Rule T8: Process recursion

Process 'calls' are simply translated by referencing the mCRL2 counterpart of the named AWN process that is being 'called' and providing it with the translation of each of the parameter values that were provided in AWN:

$$\mathrm{T}_V(\xi, \mathrm{X}(\mathit{exp}_1, \cdots, \mathit{exp}_n)) = \mathrm{X}(\mathrm{T}_\xi(\mathit{exp}_1), \cdots, \mathrm{T}_\xi(\mathit{exp}_n)) \quad \text{T8}$$
$$\text{where } \mathrm{T}(\mathrm{X}(\texttt{var}_1, \cdots, \texttt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}) = \mathrm{X}(\mathrm{T}(\texttt{var}_1) : \mathrm{sort}(\mathrm{T}_\xi(\mathit{exp}_1)), \cdots, \mathrm{T}(\texttt{var}_n) : \mathrm{sort}(\mathrm{T}_\xi(\mathit{exp}_n))) \stackrel{\text{def}}{=} \mathrm{T}_{\{\texttt{var}_1, \cdots, \texttt{var}_n\}}(\emptyset, \mathrm{p})$$

Rule T9: Choice

A choice between two branches in AWN is translated directly to a choice between two summands in mCRL2:

$$\mathrm{T}_V(\xi, \mathrm{p} + \mathrm{q}) = \mathrm{T}_V(\xi, \mathrm{p}) + \mathrm{T}_V(\xi, \mathrm{q}) \quad \text{T9}$$

Rule T10: Guard

$$\mathrm{T}_V(\xi, [\phi]\mathrm{p}) = \sum_{\mathrm{T}(\mathrm{Fv}(\phi)\ \setminus\ V)} \mathrm{T}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \texttt{msg}_{\mathit{dummy}}).\mathrm{T}_{V\ \cup\ \mathrm{Fv}(\phi)}(\xi, \mathrm{p}) \quad \text{T10}$$

The exclusive purpose of carrying around the set of assigned variables V is to determine which variables are assigned as part of a guard action: any variable that is free in the guard condition ϕ and *not* in V (and is therefore unassigned at the moment of execution) will be added as a quantifier to the $\sum$ operator. Obviously, this means that the subsequent process p must find $\mathrm{Fv}(\phi)$ – all free variables in ϕ – in its index parameter V.
One might wonder about the three placeholder parameters of the **t**, or indeed about the reason why **t** was used rather than τ. The answers to these riddles are given in the description of translation rule T7.

Rule T11: Process definition

$$\mathrm{T}(\mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}) = \mathrm{X}(\mathrm{T}(\mathtt{var}_1) : \mathrm{sort}(\mathrm{T}(\mathtt{var}_1)), \cdots, \mathrm{T}(\mathtt{var}_n) : \mathrm{sort}(\mathrm{T}(\mathtt{var}_n))) \stackrel{\text{def}}{=} \mathrm{T}_{\{\mathtt{var}_1, \cdots, \mathtt{var}_n\}}(\emptyset, \mathrm{p}) \quad \text{T11}$$

The translation of an AWN process definition X has the same name in mCRL2. Furthermore, each parameter of the mCRL2 definition is a translated parameter of X, and the body of the mCRL2 process definition is the translated body of X. When the body of X is translated, it is assumed that all process parameters are in the set of assigned variables, V, and that the valuation ξ is empty (a process definition 'knows' that the process parameters are assigned, but not what the semantic values of the process parameters are – this is established at execution time).

3.2 Translating higher-level process expressions

The rules for translating sequential process expressions can be found in Table 2.5.

$$\mathrm{T}(\xi, \mathrm{p}) = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \quad \text{T12}$$

$$\mathrm{T}(\mathrm{P} \lll \mathrm{Q}) = \nabla_V \Gamma_{\{\mathtt{r}|\mathtt{s}\to\mathtt{t}\}}(\rho_{\{\textbf{receive}\to\mathtt{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\textbf{send}\to\mathtt{s}\}}\mathrm{T}(\mathrm{Q})) \text{ where } V = \{\mathbf{t}, \mathbf{cast}, \neg\mathbf{uni}, \mathbf{send}, \mathbf{del}, \mathbf{receive}\} \quad \text{T13}$$

$$\begin{aligned}
\mathrm{T}(\mathit{ip} : \mathrm{P} : R) &= \nabla_V \Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(\mathit{ip})}, t_{\mathrm{U}(R)})) \quad \text{T14}\\
&\text{where } V = \{\mathbf{t}, \mathbf{starcast}, \mathbf{arrive}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}\\
&\text{where } C = \{\mathbf{cast}|\overline{\mathbf{cast}} \to \mathbf{starcast}, \neg\mathbf{uni}|\overline{\neg\mathbf{uni}} \to \mathbf{t}, \mathbf{del}|\overline{\mathbf{del}} \to \mathbf{deliver}, \mathbf{receive}|\overline{\mathbf{receive}} \to \mathbf{arrive}\}\\
\text{where } \mathrm{G}(\mathtt{ip}, \mathtt{R}) &= \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(\mathtt{R} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{d}:\mathrm{IP},\mathtt{msg}:\mathrm{MSG}}(\mathtt{d} \notin \mathtt{R}) \to \overline{\neg\mathbf{uni}}(\{\mathtt{d}\}, \emptyset, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{data}:\mathrm{DATA}} \overline{\mathbf{del}}(\mathtt{ip}, \mathtt{data}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(\mathtt{ip} \in \mathtt{D'}) \to \overline{\mathbf{receive}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(\mathtt{ip} \notin \mathtt{D'}) \to \mathbf{arrive}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(\mathtt{ip}, \mathtt{ip'}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \cup \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(\mathtt{ip'}, \mathtt{ip}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \cup \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'},\mathtt{ip''}:\mathrm{IP}}(\mathtt{ip} \notin \{\mathtt{ip'}, \mathtt{ip''}\}) \to \mathbf{connect}(\mathtt{ip'}, \mathtt{ip''}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{disconnect}(\mathtt{ip}, \mathtt{ip'}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \setminus \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{disconnect}(\mathtt{ip'}, \mathtt{ip}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \setminus \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'},\mathtt{ip''}:\mathrm{IP}}(\mathtt{ip} \notin \{\mathtt{ip'}, \mathtt{ip''}\}) \to \mathbf{disconnect}(\mathtt{ip'}, \mathtt{ip''}).\mathrm{G}(\mathtt{ip}, \mathtt{R})
\end{aligned}$$

$$\begin{aligned}
\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) &= \rho_R \nabla_V \Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\mathbf{a}\}} \Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \quad \text{T15}\\
&\text{where } R = \{\mathbf{a} \to \mathbf{arrive}, \mathbf{c} \to \mathbf{connect}, \mathbf{d} \to \mathbf{disconnect}, \mathbf{s} \to \mathbf{starcast}\}\\
&\text{where } V = \{\mathbf{a}, \mathbf{c}, \mathbf{d}, \mathbf{deliver}, \mathbf{s}, \mathbf{t}\}\\
&\text{where } C = \{\mathbf{starcast}|\mathbf{arrive} \to \mathbf{s}, \mathbf{connect}|\mathbf{connect} \to \mathbf{c}, \mathbf{disconnect}|\mathbf{disconnect} \to \mathbf{d}\}
\end{aligned}$$

$$\begin{aligned}
\mathrm{T}([\ \mathrm{M}\ ]) &= \nabla_V \rho_{\{\textbf{starcast}\to\mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \quad \text{T16}\\
&\text{where } V = \{\mathbf{t}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}\\
&\text{where } C = \{\overline{\mathbf{newpkt}}|\mathbf{arrive} \to \mathbf{newpkt}\}\\
&\text{where } \mathrm{H} = \textstyle\sum_{\mathtt{ip}:\mathrm{IP},\mathtt{data}:\mathrm{DATA},\mathtt{dest}:\mathrm{IP}} \overline{\mathbf{newpkt}}(\{\mathtt{ip}\}, \{\mathtt{ip}\}, \mathtt{newpkt}(\mathtt{data}, \mathtt{dest})).\mathrm{H}
\end{aligned}$$

Table 2.5: *Rules for translating AWN's higher-level process expressions*

The translation rules in Table 2.5 use $\mathrm{T}(\mathrm{p})$ as the signature of the translation function, where p is the process expression to be translated. Clearly, $\mathrm{T}(\mathrm{p})$ no longer has the ξ parameter (ξ, p in rule T12 forms a single parameter, the state of a sequential process in AWN to be precise), and it does not carry a set of assigned variables V anymore. At this level of the AWN specification, syntactic data expressions have disappeared.
The following subsections will elaborate on each of the translation rules in Table 2.5.

Rule T12: Sequential process

This rule allows the rules from Table 2.5 to make use of the rules in Table 2.4:

$$\mathrm{T}(\xi, \mathrm{p}) = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \quad \text{T12}$$

Note that ξ, p forms a single parameter – namely the state of a sequential process in AWN – and that this state is separated into its two components, the valuation ξ and the process syntax p. Furthermore, the set of assigned variables V is assigned with the domain of ξ (in the implementation, this is almost never of any consequence; when proving the correctness of $\mathrm{T}(\mathrm{p})$, however, this becomes a different story).

Rule T13: Parallel processes

$$\mathrm{T}(\mathrm{P} \between \mathrm{Q}) = \nabla_V \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{t}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \text{ where } V = \{\mathbf{t}, \mathbf{cast}, \neg\mathbf{uni}, \mathbf{send}, \mathbf{del}, \mathbf{receive}\} \quad \text{T13}$$

Two AWN processes in parallel are translated by applying several mCRL2 operators in sequence:

1. It must be possible in the final mCRL2 operator to differentiate between the **receive** actions that occur in $\mathrm{T}(\mathrm{P})$ and those that occur in $\mathrm{T}(\mathrm{Q})$, as well as between the **send** actions that occur in $\mathrm{T}(\mathrm{P})$ and those that occur in $\mathrm{T}(\mathrm{Q})$. The **receive** actions of $\mathrm{T}(\mathrm{P})$ are therefore renamed to **r** and the **send** actions of $\mathrm{T}(\mathrm{Q})$ to **s**.
2. The resulting processes are put in parallel.
3. Occurrences of **r**|**s** action labels are replaced by a **t** action label (this is why **t** has been given three parameters).
4. Only a specific selection of actions is allowed to occur. This blocks all mCRL2 multi-actions that are impossible in AWN (such as **t**|**t**). In addition, all remaining **r** actions are blocked (from now on, $\mathrm{T}(\mathrm{P})$ can only **receive** messages if they arrive via $\mathrm{T}(\mathrm{Q})$, but $\mathrm{T}(\mathrm{Q})$ can still **receive**) as well as all remaining **s** actions (**send** actions of $\mathrm{T}(\mathrm{Q})$ must have synchronized before now, but $\mathrm{T}(\mathrm{P})$ can still **send**).

Rule T14: Node

$$\begin{aligned}
\mathrm{T}(ip : \mathrm{P} : R) &= \nabla_V \Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{U(ip)}, t_{U(R)})) \quad \text{T14}\\
&\text{where } V = \{\mathbf{t}, \mathbf{starcast}, \mathbf{arrive}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}\\
&\text{where } C = \{\mathbf{cast}|\overline{\mathbf{cast}} \to \mathbf{starcast}, \neg\mathbf{uni}|\overline{\neg\mathbf{uni}} \to \mathbf{t}, \mathbf{del}|\overline{\mathbf{del}} \to \mathbf{deliver}, \mathbf{receive}|\overline{\mathbf{receive}} \to \mathbf{arrive}\}\\
\text{where } \mathrm{G}(\mathtt{ip}, \mathtt{R}) &= \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (\mathtt{R} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{d}:\mathrm{IP},\mathtt{msg}:\mathrm{MSG}} (\mathtt{d} \notin \mathtt{R}) \to \overline{\neg\mathbf{uni}}(\{\mathtt{d}\}, \emptyset, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{data}:\mathrm{DATA}} \overline{\mathbf{del}}(\mathtt{ip}, \mathtt{data}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (\mathtt{ip} \in \mathtt{D'}) \to \overline{\mathbf{receive}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (\mathtt{ip} \notin \mathtt{D'}) \to \mathbf{arrive}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(\mathtt{ip}, \mathtt{ip'}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \cup \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(\mathtt{ip'}, \mathtt{ip}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \cup \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'},\mathtt{ip''}:\mathrm{IP}} (\mathtt{ip} \notin \{\mathtt{ip'}, \mathtt{ip''}\}) \to \mathbf{connect}(\mathtt{ip'}, \mathtt{ip''}).\mathrm{G}(\mathtt{ip}, \mathtt{R})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{disconnect}(\mathtt{ip}, \mathtt{ip'}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \setminus \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{disconnect}(\mathtt{ip'}, \mathtt{ip}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \setminus \{\mathtt{ip'}\})\\
&+ \textstyle\sum_{\mathtt{ip'},\mathtt{ip''}:\mathrm{IP}} (\mathtt{ip} \notin \{\mathtt{ip'}, \mathtt{ip''}\}) \to \mathbf{disconnect}(\mathtt{ip'}, \mathtt{ip''}).\mathrm{G}(\mathtt{ip}, \mathtt{R})
\end{aligned}$$

This rule elevates processes to the network level by synchronizing them with a process G that provides the address of the node on which they run and the set of addresses of nodes that are within transmission range. Through this synchronization, the actions **cast**, ¬**uni**, **del**, and **receive** of a process $\mathrm{T}(\mathrm{P})$ are forced to have certain parameter values, which eliminates many – but not all – of the superfluous mCRL2 actions that are impossible in AWN. G also adds independent node behavior, namely **arrive** (which allows a node to synchronize the arrival of messages at other nodes with the rest of the network), **connect**, and **disconnect** actions.
At the end, only single actions with certain labels are permitted to occur: multi-actions produced by putting $\mathrm{T}(\mathrm{P})$ and G in parallel are blocked, and so are leftover **cast**, $\overline{\mathbf{cast}}$, ¬**uni**, $\overline{\neg\mathbf{uni}}$, **del**, $\overline{\mathbf{del}}$, **receive**, and $\overline{\mathbf{receive}}$ actions.

Rule T15: Parallel nodes

$$\begin{aligned}&\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) = \rho_R \nabla_V \Gamma_{\{\textbf{arrive}\mid\textbf{arrive}\rightarrow\textbf{a}\}} \Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \quad \text{T15}\\ &\text{where } R = \{\textbf{a} \rightarrow \textbf{arrive}, \textbf{c} \rightarrow \textbf{connect}, \textbf{d} \rightarrow \textbf{disconnect}, \textbf{s} \rightarrow \textbf{starcast}\}\\ &\text{where } V = \{\textbf{a}, \textbf{c}, \textbf{d}, \textbf{deliver}, \textbf{s}, \textbf{t}\}\\ &\text{where } C = \{\textbf{starcast}\mid\textbf{arrive} \rightarrow \textbf{s}, \textbf{connect}\mid\textbf{connect} \rightarrow \textbf{c}, \textbf{disconnect}\mid\textbf{disconnect} \rightarrow \textbf{d}\}\end{aligned}$$

This rule produces a sequence of mCRL2 operators that restricts the behavior of $\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})$ in such a way that any remaining action that can occur has a possible counterpart in AWN. In particular, the sequence of mCRL2 operators enforces that:

- One process does **starcast** action *and* the other process does a matching **arrive** action;
- Both processes do the same **connect** or **disconnect** action;
- One of the processes does a **deliver** action and the other process does nothing.

Rule T16: Network

$$\begin{aligned}&\mathrm{T}([\ \mathrm{M}\ ]) = \nabla_V \rho_{\{\textbf{starcast}\rightarrow\textbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \quad \text{T16}\\ &\text{where } V = \{\textbf{t}, \textbf{newpkt}, \textbf{deliver}, \textbf{connect}, \textbf{disconnect}\}\\ &\text{where } C = \{\overline{\textbf{newpkt}}\mid\textbf{arrive} \rightarrow \textbf{newpkt}\}\\ &\text{where } \mathrm{H} = \textstyle\sum_{\mathtt{ip:IP},\mathtt{data:DATA},\mathtt{dest:IP}} \overline{\textbf{newpkt}}(\{\mathtt{ip}\},\{\mathtt{ip}\},\mathtt{newpkt}(\mathtt{data},\mathtt{dest})).\mathrm{H}\end{aligned}$$

At the highest level of the AWN specification, it is possible to inject messages into the network via **newpkt** actions. In mCRL2, this behavior is enabled by translation rule T16. A **newpkt** action is generated from a remaining **arrive** action (which means that all nodes in the network are taking this action in parallel) that has exactly one node both as intended and as actual destination. In addition, the message parameter of the action is forced to have a specific `newpkt` format.
The rule also renames remnant **starcast** actions to **t** and finally blocks multi-actions such as **connect**|$\overline{\textbf{newpkt}}$ as well as leftover **arrive** and $\overline{\textbf{newpkt}}$ actions.

3.3 Totalness

This section shows that the translation function T is total.

> **Lemma 3.1** The translation function T as defined via the rules in Tables 2.4 and 2.5 is *total*; that is, $\mathrm{T}(\mathrm{P})$ is defined for all valid AWN expressions P.

Proof. The proof is trivial, and therefore given informally.
First, there must exist a matching translation rule for each rule of the grammar of AWN. This is indeed the case.
Second, there must exist a matching translation rule for each input at recursive applications of T. Considering that recursive applications only receive process expressions from the original AWN process expression as input, such as p or q, these process expressions must match the grammar of AWN, and therefore a matching translation rule must exist.
Third, there should be no infinite recursion. Since the input at recursive applications of T is always a strictly smaller expression than the input of the enveloping translation rule, recursion is always finite.
Finally, $\mathrm{T}_\xi(e)$ must be defined for all valid data expressions e. Combining property (i) of $\mathrm{T}_\xi(e)$ with the fact that $\forall e \in M_{\mathrm{sort}(e)} \,.\, \exists t \,.\, [\![t]\!] = e$ (Definition 15.2.17 from the mCRL2 book) where M_D is the set of all semantic values of a sort D, it follows that T_ξ is a total function. □

3.4 Translation relation

Proving the correctness of the translation function involves showing the existence of a strong bisimulation between an AWN process and its translated counterpart in mCRL2. Because a strong bisimulation is a relation, it is useful to express the translation function as relation:

$$\widetilde{\mathrm{T}} \stackrel{\text{def}}{=} \{\ (\mathrm{P}, \mathrm{T}(\mathrm{P})) \mid \mathrm{P} \text{ is an AWN parallel process expression, node expression, or (partial) network expression } \} \tag{2.1}$$

Ultimately, however, the correctness proof will be a statement about the following relation:

$$\widetilde{\mathrm{T}}_\tau \stackrel{\text{def}}{=} \left\{\ ([\![\ \mathrm{M}\ ]\!], \tau_{\{\mathrm{t}\}}\mathrm{T}([\![\ \mathrm{M}\ ]\!])) \;\middle|\; [\![\ \mathrm{M}\ ]\!] \text{ is an AWN network expression } \right\} \tag{2.2}$$

According to Lemma 3.1, both relations are defined for all valid AWN expressions P.

3.5 Action relation

The translation function T changes AWN actions to mCRL2 actions in a manner that is not always straightforward. To formally prove the correctness of T, the precise relation between AWN actions and mCRL2 actions must be made explicit first. The relation is called $\mathcal{A}$ and its definition can be found in Table 2.6.

$$\begin{aligned}
\mathcal{A} \stackrel{\text{def}}{=} \{\ & (\{\ \tau\ \}, \{\ \mathbf{t}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))\ \}), && (2.3)\\
& (\{\ \mathbf{broadcast}(\xi(ms))\ \}, \{\ \mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])\ |\ \hat{\mathtt{D}} : \mathrm{Set(IP)}\ \}), && (2.4)\\
& (\{\ \mathbf{groupcast}(\xi(dests), \xi(ms))\ \}, \{\ \mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])\ |\ \hat{\mathtt{D}} : \mathrm{Set(IP)}\ \}), && (2.5)\\
& (\{\ \mathbf{unicast}(\xi(dest), \xi(ms))\ \}, \{\ \mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \}), && (2.6)\\
& (\{\ \neg\mathbf{unicast}(\xi(dest), \xi(ms))\ \}, \{\ \neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \}), && (2.7)\\
& (\{\ \mathbf{send}(\xi(ms))\ \}, \{\ \mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \}), && (2.8)\\
& (\{\ \mathbf{deliver}(\xi(data))\ \}, \{\ \mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])\ |\ \hat{\mathtt{i}} : \mathrm{IP}\ \}), && (2.9)\\
& (\{\ \mathbf{receive}(m)\ \}, \{\ \mathbf{receive}([\![\hat{\mathtt{D}}]\!], [\![\hat{\mathtt{D}}\mathtt{'}]\!], \mathrm{U}(m))\ |\ \hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'} : \mathrm{Set(IP)}\ \}), && (2.10)\\
& (\{\ R{:}\mathbf{*cast}(m)\ \}, \{\ \mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))\ \}), && (2.11)\\
& (\{\ ip{:}\mathbf{deliver}(d)\ \}, \{\ \mathbf{deliver}(\mathrm{U}(ip), \mathrm{U}(d))\ \}), && (2.12)\\
& (\{\ H\neg K{:}\mathbf{arrive}(m)\ \}, \left\{\ \mathbf{arrive}([\![\hat{\mathtt{D}}]\!], [\![\hat{\mathtt{D}}\mathtt{'}]\!], \mathrm{U}(m)) \;\middle|\; \begin{smallmatrix} \hat{\mathtt{D}},\hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)} \\ \hat{\mathtt{D}}\mathtt{'}\subseteq\hat{\mathtt{D}} \\ H\subseteq\hat{\mathtt{D}}\mathtt{'} \\ K\cap\hat{\mathtt{D}}\mathtt{'}=\emptyset \end{smallmatrix} \right\}), && (2.13)\\
& (\{\ \mathbf{connect}(ip\mathtt{'}, ip\mathtt{''})\ \}, \{\ \mathbf{connect}(\mathrm{U}(ip\mathtt{'}), \mathrm{U}(ip\mathtt{''}))\ \}), && (2.14)\\
& (\{\ \mathbf{disconnect}(ip\mathtt{'}, ip\mathtt{''})\ \}, \{\ \mathbf{disconnect}(\mathrm{U}(ip\mathtt{'}), \mathrm{U}(ip\mathtt{''}))\ \}), && (2.15)\\
& (\{\ ip{:}\mathbf{newpkt}(d, dip)\ \}, \{\ \mathbf{newpkt}(\{\mathrm{U}(ip)\}, \{\mathrm{U}(ip)\}, \mathtt{newpkt}(\mathrm{U}(d), \mathrm{U}(dip)))\ \}) && (2.16)\\
\mid\ & m, \xi(ms) : \mathrm{MSG};\quad ip, ip\mathtt{'}, ip\mathtt{''}, dip, dest : \mathrm{IP};\quad d, \xi(data) : \mathrm{DATA};\quad dests, R, D, H, K : \mathrm{Set(IP)};\quad R \subseteq D\quad \}
\end{aligned}$$

Table 2.6: *Action relation $\mathcal{A}$*

4 Correctness proof

4.1 Overview

TODO

4.2 Relation definitions

Definition 4.1 (Strong simulation) Let $\mathrm{LTS}_1 = (S_1, \mathrm{Act}, \rightarrow_1)$ and $\mathrm{LTS}_2 = (S_2, \mathrm{Act}, \rightarrow_2)$ be labeled transition systems where $\rightarrow_i \subseteq S_i \times \mathrm{Act} \times S_i$. Then $R \subseteq S_1 \times S_2$ is called a *strong simulation of* LTS_1 *by* LTS_2 if and only if

$$\forall (\mathrm{p}, \mathrm{q}) \in R, a \in \mathrm{Act}, \mathrm{p}' \in S_1 \,.\, \mathrm{p} \xrightarrow{a}_1 \mathrm{p}' \Rightarrow \exists \mathrm{q}' \in S_2 \,.\, \mathrm{q} \xrightarrow{a}_2 \mathrm{q}' \wedge (\mathrm{p}', \mathrm{q}') \in R$$

Definition 4.2 (Strong bisimulation) Let $\mathrm{LTS}_1 = (S_1, \mathrm{Act}, \rightarrow_1)$ and $\mathrm{LTS}_2 = (S_2, \mathrm{Act}, \rightarrow_2)$ be labeled transition systems where $\rightarrow_i \subseteq S_i \times \mathrm{Act} \times S_i$. Then $R \subseteq S_1 \times S_2$ is called a *strong bisimulation between* LTS_1 *and* LTS_2 if and only if

- R is a strong simulation of LTS_1 by LTS_2 and
- $R^\smile$ is a strong simulation of LTS_2 by LTS_1.

Definition 4.3 Let $\mathrm{LTS}_1 = (S_1, \mathrm{Act}_1, \rightarrow_1)$ and $\mathrm{LTS}_2 = (S_2, \mathrm{Act}_2, \rightarrow_2)$ be labeled transition systems where $\rightarrow_i \subseteq S_i \times \mathrm{Act} \times S_i$ and let $\mathfrak{f} : \mathrm{Act}_1 \rightarrow \mathrm{Act}_2$ be a bijective function. Then $R \subseteq S_1 \times S_2$ is called a *strong simulation of* LTS_1 *by* LTS_2 *modulo renaming by* $\mathfrak{f}$ if and only if

$$\forall (\mathrm{p}, \mathrm{q}) \in R, a \in \mathrm{Act}, \mathrm{p}' \in S_1 \,.\, \mathrm{p} \xrightarrow{a}_1 \mathrm{p}' \Rightarrow \exists \mathrm{q}' \in S_2 \,.\, \mathrm{q} \xrightarrow{\mathfrak{f}(a)}_2 \mathrm{q}' \wedge (\mathrm{p}', \mathrm{q}') \in R$$

Definition 4.4 Let $\mathrm{LTS}_1 = (S_1, \mathrm{Act}_1, \rightarrow_1)$ and $\mathrm{LTS}_2 = (S_2, \mathrm{Act}, \rightarrow_2)$ be labeled transition systems where $\rightarrow_i \subseteq S_i \times \mathrm{Act} \times S_i$ and let $\mathfrak{f} : \mathrm{Act}_1 \rightarrow \mathrm{Act}_2$ be a bijective function. Then $R \subseteq S_1 \times S_2$ is called a *strong bisimulation between* LTS_1 *and* LTS_2 *modulo renaming by* $\mathfrak{f}$ if and only if

- R is a strong simulation of LTS_1 by LTS_2 modulo renaming by $\mathfrak{f}$ and
- $R^\smile$ is a strong simulation of LTS_2 by LTS_1 modulo renaming by $\mathfrak{f}^{-1}$.

Definition 4.5 Let $\mathrm{LTS}_1 = (S_1, \mathrm{Act}_1, \rightarrow_1)$ and $\mathrm{LTS}_2 = (S_2, \mathrm{Act}_2, \rightarrow_2)$ be labeled transition systems where $\rightarrow_i \subseteq S_i \times \mathrm{Act}_i \times S_i$ and let $\mathcal{A} \subseteq \mathcal{P}(\mathrm{Act}_1) \times \mathcal{P}(\mathrm{Act}_2)$. Then $R \subseteq S_1 \times S_2$ is a *strong $\mathcal{A}$-warped simulation of* LTS_1 *by* LTS_2 if and only if

$$\forall (\mathrm{p}, \mathrm{q}) \in R, a \in \mathrm{Act}_1, \mathrm{p}' \in S_1 \,.\, \Big(\mathrm{p} \xrightarrow{a} \mathrm{p}' \Rightarrow \exists (A_1, A_2) \in \mathcal{A} \,.\, a \in A_1 \wedge \exists \mathrm{q}' \in S_2 \,.\, \mathrm{p} \xrightarrow{A_1} \mathrm{p}' \wedge \mathrm{q} \xrightarrow{A_2} \mathrm{q}' \wedge (\mathrm{p}', \mathrm{q}') \in R\Big)$$

$$\text{where } \mathrm{p} \xrightarrow{A} \mathrm{p}' \Leftrightarrow \forall a \in A \,.\, \mathrm{p} \xrightarrow{a} \mathrm{p}'$$

Definition 4.6 Let $\mathrm{LTS}_1 = (S_1, \mathrm{Act}_1, \rightarrow_1)$ and $\mathrm{LTS}_2 = (S_2, \mathrm{Act}_2, \rightarrow_2)$ be labeled transition systems where $\rightarrow_i \subseteq S_i \times \mathrm{Act}_i \times S_i$ and let $\mathcal{A} \subseteq \mathcal{P}(\mathrm{Act}_1) \times \mathcal{P}(\mathrm{Act}_2)$. Then $R \subseteq S_1 \times S_2$ is called a *strong $\mathcal{A}$-warped bisimulation between* LTS_1 *and* LTS_2 if and only if

- R is a strong $\mathcal{A}$-warped simulation of LTS_1 by LTS_2 and
- $R^\smile$ is a strong $\mathcal{A}^\smile$-warped simulation of LTS_2 by LTS_1.

4.3 Auxiliary lemmas

Definition 4.7 In the context of this document, a *one-way step* is a step in an mCRL2 derivation

- that requires no premises, or
- that has more than one premise, or

- that has one premise and changes the process expression on the left side of the arrow of its premise to a process expression that is *not* semantically equivalent.

The number of one-way steps in an mCRL2 derivation is limited and when their ordering is changed, the original conclusion of the derivation can no longer be reached. All inference rules in mCRL2 are one-way steps. **(*Rewritten.*)**

Definition 4.8 In the context of this document, a *rewrite step* is a step in an mCRL2 derivation with exactly one premise that produces a conclusion that is semantically equivalent to its premise.
Rewrite steps can be applied in both directions (starting from the premise or starting from the conclusion) making every rewrite step reversible. It is therefore impossible to cause the original conclusion of a derivation to become unreachable by the insertion of rewrite steps. **(*Rewritten.*)**

Definition 4.9 In the context of this document, a *representative derivation* is a derivation in mCRL2 that describes an entire class of derivations in mCRL2 in a generalized manner. A representative derivation describes all derivations that

- only consist of one-way steps (see Definition 4.7) and rewrite steps (see Definition 4.8), and
- have the same ordering of one-way steps.

Note that derivations in mCRL2 can only consist of one-way steps (the inference rules) and rewrite steps (rewriting of syntax based on definitions and lemmas). There therefore exists a representative derivation for every mCRL2 derivation. As a consequence of the second property, the conclusion of a representative derivation is semantically equivalent to the conclusions of the derivations that it describes.
Finally, given a representative derivation with conclusion $\mathrm{P} \xrightarrow{a} \mathrm{Q}$, this document may state that the derivation is 'without alternatives': this means that there does not exist a derivation with different or differently ordered one-way steps that results in a conclusion $\mathrm{P}' \xrightarrow{a'} \mathrm{Q}'$ such that P and P' are semantically equivalent. This is useful for exhaustively listing the behavior of process expression P.

Several lemmas related to τ_ξ are used for the correctness proof (see Section 4) which are listed below.

Lemma 4.1 If $\xi(exp)$ is defined, then $\tau_\xi(exp) = t_{\mathrm{U}(\xi(exp))}$.

Proof. First note that because $\xi(exp)$ is defined, it must be the case that exp is closed under ξ; that is, $\mathrm{Fv}(exp) \subseteq \mathrm{DOM}(\xi)$. From Property (iii) and the definition of τ_ξ it then follows that $\mathrm{Fv}(\tau(exp)\left[\tau(\mathtt{x}) := t_{\mathrm{U}(y)} \mid \mathtt{x} := y \in \xi\right]) = \emptyset$. Consequently,

$$\begin{aligned}
\tau_\xi(exp) &\overset{\text{Definition of } \tau_\xi}{=} \mathcal{F}\left(\tau(exp)\left[\tau(\mathtt{x}) := t_{\mathrm{U}(y)} \mid \mathtt{x} := y \in \xi\right]\right) \\
&\overset{\text{Definition of } \mathcal{F}}{=} t_{[\![\tau(exp)\left[\tau(\mathtt{x}) := t_{\mathrm{U}(y)} \mid \mathtt{x} := y \in \xi\right]]\!]} \\
&\overset{\text{Definition of } \tau_\xi}{=} t_{[\![\tau_\xi(exp)]\!]} \\
&\overset{\text{Property (ii)}}{=} t_{\mathrm{U}(\xi(exp))}
\end{aligned}$$

□

Lemma 4.2 If e is a semantic AWN value and $\mathtt{v} \notin \mathrm{DOM}(\xi)$, then $\tau_\xi(exp)\left[\tau(\mathtt{v}) := t_{\mathrm{U}(e)}\right] = \tau_{\xi[\mathtt{v}:=e]}(exp)$.

Proof.

$$\begin{aligned}
\tau_\xi(exp)\left[\tau(\mathtt{v}) := t_{\mathrm{U}(e)}\right] &\overset{\text{Definition of } \tau_\xi}{=} \tau(exp)\left[\tau(\mathtt{x}) := t_{\mathrm{U}(y)} \mid \mathtt{x} := y \in \xi\right]\left[\tau(\mathtt{v}) := t_{\mathrm{U}(e)}\right] \\
&\overset{\mathtt{v} \notin \mathrm{DOM}(\xi)}{=} \tau(exp)\left[\tau(\mathtt{x}) := t_{\mathrm{U}(y)} \mid \mathtt{x} := y \in \xi[\mathtt{v} := e]\right] \\
&\overset{\text{Definition of } \tau_\xi}{=} \tau_{\xi[\mathtt{v}:=e]}(exp)
\end{aligned}$$

□

Lemma 4.3 If e is a semantic AWN value and $\mathtt{v} \notin \textsc{dom}(\xi)$, then $\mathrm{T}_V(\xi, \mathrm{p})\left[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}\right] = \mathrm{T}_V(\xi[\mathtt{v} := e], \mathrm{p})$.

Proof. Proof by structural induction:
Base case is T8, which is trivial (use Lemma 4.2).
Induction step is divided into cases for translation rules T1 to T7 plus T9 and T10. These translation rules kinda look like this:

$$\mathrm{T}_V(\xi, \mathbf{a}(e_1, \cdots, e_n).\mathrm{p}) = \sum \mathbf{b}(\mathrm{T}_\xi(e_1), \cdots, \mathrm{T}_\xi(e_n)).\mathrm{T}_{V'}(\xi, \mathrm{p})$$

Using this template, the individual cases would look something like this:

$$\begin{aligned}
&\mathrm{T}_V(\xi, \mathbf{a}(e_1, \cdots, e_n).\mathrm{p})[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}] \\
&\overset{\text{Definition of T}}{=} (\textstyle\sum \mathbf{b}(\mathrm{T}_\xi(e_1), \cdots, \mathrm{T}_\xi(e_n)).\mathrm{T}_{V'}(\xi, \mathrm{p}))[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}] \\
&= \textstyle\sum \mathbf{b}(\mathrm{T}_\xi(e_1), \cdots, \mathrm{T}_\xi(e_n))[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}].\mathrm{T}_{V'}(\xi, \mathrm{p})[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}] \\
&= \textstyle\sum \mathbf{b}(\mathrm{T}_\xi(e_1)[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}], \cdots, \mathrm{T}_\xi(e_n)[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}]).\mathrm{T}_{V'}(\xi, \mathrm{p})[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}] \\
&\overset{\text{Lemma 4.2}}{=} \textstyle\sum \mathbf{b}(\mathrm{T}_{\xi[\mathtt{v}:=e]}(e_1), \cdots, \mathrm{T}_{\xi[\mathtt{v}:=e]}(e_n)).\mathrm{T}_{V'}(\xi, \mathrm{p})[\mathrm{T}(\mathtt{v}) := t_{\mathrm{U}(e)}] \\
&\overset{\text{I.H.}}{=} \textstyle\sum \mathbf{b}(\mathrm{T}_{\xi[\mathtt{v}:=e]}(e_1), \cdots, \mathrm{T}_{\xi[\mathtt{v}:=e]}(e_n)).\mathrm{T}_{V'}(\xi[\mathtt{v} := e], \mathrm{p}) \\
&\overset{\text{Definition of T}}{=} \mathrm{T}_V(\xi[\mathtt{v} := e], \mathbf{a}(e_1, \cdots, e_n).\mathrm{p})
\end{aligned}$$

□

The following lemma is also used:

Lemma 4.4 If e is a semantic AWN value, then $[\![t_{\mathrm{U}(e)}]\!] = \mathrm{U}(e)$.

Proof. You need a proof for this? Really? □

Lemma 4.5 If e is a semantic AWN value and $\xi(exp)$ is defined, then $\mathrm{U}(e) = [\![\mathrm{T}_\xi(exp)]\!] \Leftrightarrow e = \xi(exp)$.

Proof.

$$\left.\begin{array}{c} \mathrm{U}(e) \overset{\text{Lemma 4.4}}{=} [\![t_{\mathrm{U}(e)}]\!] \\ [\![\mathrm{T}_\xi(exp)]\!] \overset{\text{Lemma 4.1}}{=} [\![t_{\mathrm{U}(\xi(exp))}]\!] \end{array}\right\} \Leftrightarrow e = \xi(exp)$$

□

Definition 4.10 See Definition 15.2.12 in the mCRL2 book.

4.4 Proof

Lemma 4.6 Take translation relation $\widetilde{\mathrm{T}}$ from Table **??** and action relation $\mathcal{A}$ from Table 2.6. Then $\widetilde{\mathrm{T}}$ is a strong $\mathcal{A}$-warped simulation of AWN expressions P by mCRL2 expressions $\mathrm{T}(\mathrm{P})$ for all AWN expressions P.

Proof. Using Lemma 3.1, for Definition 4.5 to apply it is sufficient to prove that

$$\forall \mathrm{P}, \mathrm{P}', a \,.\, \Big(\mathrm{P} \xrightarrow{a} \mathrm{P}' \Rightarrow \exists (A_1, A_2) \in \mathcal{A} \,.\, a \in A_1 \wedge \mathrm{P} \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_2} \mathrm{T}(\mathrm{P}') \Big) \tag{2.17}$$

The proof of Equation 2.17 is by structural induction over the inference rules of AWN.
Induction hypothesis: For all premises $\mathrm{P} \xrightarrow{a} \mathrm{P}'$ of an AWN inference rule, it holds that

$$\exists (A_1, A_2) \in \mathcal{A} \,.\, a \in A_1 \wedge \mathrm{P} \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_2} \mathrm{T}(\mathrm{P}')$$

Base cases: Show for each axiomatic inference rule of AWN with conclusion $\mathrm{P} \xrightarrow{a} \mathrm{P}'$ that there is some $(A_1, A_2) \in \mathcal{A} \,.\, a \in A_1 \wedge \mathrm{P} \xrightarrow{A_1} \mathrm{P}'$ such that $\mathrm{T}(\mathrm{P}) \xrightarrow{a'} \mathrm{T}(\mathrm{P}')$ can be derived for all $a' \in A_2$.

Example: AWN defines the inference rule

$$\frac{}{\xi, \mathbf{broadcast}(ms).\mathrm{p} \xrightarrow{\mathbf{broadcast}(\xi(ms))} \xi, \mathrm{p}}\ \textsc{Broadcast (T1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{broadcast}(\xi(ms)) \in A_1 \wedge \xi, \mathbf{broadcast}(ms).\mathrm{p} \xrightarrow{A_1} \xi, \mathrm{p}$, namely Pair 2.4 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi, \mathrm{p})$ for all $a \in A_2 = \{\ \mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket) \mid \hat{\mathtt{D}} : \mathrm{Set(IP)}\ \}$.
In mCRL2, the following derivation can be made for all $\hat{\mathtt{D}}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{\llbracket \mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \rrbracket} \checkmark}\ \textsc{Axiom}}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \checkmark}\ \text{Definition 4.10}}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{Seq 1}}{(\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}))\,[\mathtt{D} := \hat{\mathtt{D}}] \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{Substitution}}{\sum_{\mathtt{D}:\mathrm{Set(IP)}} \mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{Sum 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{T1}}{\mathrm{T}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}(\xi, \mathrm{p})}\ \textsc{T12}$$

for $\hat{\mathtt{D}} : \mathrm{Set(IP)}$ and $\mathtt{D} \notin \textsc{vars}(\mathrm{T}_\xi(ms)) \cup \textsc{vars}(\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}))$. In conclusion, the induction hypothesis holds for this base case.

Induction step: Given only its side conditions and the induction hypothesis, show for each inference rule of AWN with conclusion $\mathrm{P} \xrightarrow{a} \mathrm{P}'$ that there is some $(A_1, A_2) \in \mathcal{A}\ .\ a \in A_1 \wedge \mathrm{P} \xrightarrow{A_1} \mathrm{P}'$ such that $\mathrm{T}(\mathrm{P}) \xrightarrow{a'} \mathrm{T}(\mathrm{P}')$ can be derived for all $a' \in A_2$.

Example: AWN defines the inference rule

$$\frac{\mathrm{P} \xrightarrow{\mathbf{broadcast}(m)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{R:\mathbf{*cast}(m)} ip : \mathrm{P}' : R}\ \textsc{Broadcast (T3)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $R : \mathbf{*cast}(m) \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R$, namely Pair 2.11 in Table 2.6 (note that it is possible to choose $D = \mathcal{U}_{\mathrm{IP}}$). The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R)$ for all $a \in A_2 = \{\ \mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \xrightarrow{\llbracket \overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \rrbracket} \checkmark}\ \textsc{Axiom}}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket t_{\mathrm{U}(R)} \rrbracket, \llbracket t_{\mathrm{U}(m)} \rrbracket)} \checkmark}\ \text{Definition 4.10}}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \checkmark}\ \text{Lemma 4.4}}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Seq 1}}{\llbracket t_{\mathrm{U}(R)} \cap \mathcal{U}_{\mathrm{IP}} = t_{\mathrm{U}(R)} \rrbracket = \mathit{true}}\ \textsc{Guard 2}}{(t_{\mathrm{U}(R)} \cap \mathcal{U}_{\mathrm{IP}} = t_{\mathrm{U}(R)}) \to \overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Substitution}}{(\ (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})\ )\,[\,\mathtt{D} := \mathcal{U}_{\mathrm{IP}}, \mathtt{D'} := t_{\mathrm{U}(R)}, \mathtt{msg} := t_{\mathrm{U}(m)}\,] \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Sum 2}\ \text{(3 times)}}{\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Choice 2}}{\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) + \mathrm{S}[\,\mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\,] \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Substitution}}{\text{Definition of G}\quad \Big(\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (\mathtt{R} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S}\Big)\,[\,\mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\,] \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Recursion 2}$$

where $\mathrm{S}[\,\mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\,]$ is an expression equal to all summands of G except for the first one.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))}$ because Pair 2.4 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}\ .\ \mathbf{broadcast}(m) \in B_1$. Combining this with the conclusion of the derivation above gives

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis} \quad \dfrac{}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textit{(see above)}}{\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m)) \mid \overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Par}\ 3}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\gamma_C(\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m)) \mid \overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{Comm}\ 2}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Apply}\ \gamma_C}{\rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\{\neg \mathbf{unicast} \to \mathbf{t}\} \bullet \mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{Rename}\ 2}{\mathbf{starcast} \in V \cup \{\tau\} \quad \dfrac{\rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}{\nabla_V \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \nabla_V \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{T14}}\ \textsc{Allow}\ 2}\ \text{Apply}\ \{\neg \mathbf{unicast} \to \mathbf{t}\}\bullet$$

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(ip : \mathrm{P}' : R)$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R) \text{ for all } a \in A_2 = \{\ \mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))\ \}$$

which finishes the induction step for the case of the BROADCAST (T3) inference rule.

Similar proofs must be done for all other inference rules of AWN. These proofs can be found in Appendix A. Given all of the proofs, Equation 2.17 holds. □

Lemma 4.7 For T_V as defined by the rules T1 to T10 in Table **??** it holds that

$$\forall \mathrm{p}, a, \mathrm{Q}, \xi\ .\ \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{Q} \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\smile}, \mathrm{q}, \sigma\ .\ a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q} \wedge \xi, \mathrm{p} \xrightarrow{A_2} \xi[\sigma], \mathrm{q} \wedge \mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{q}) \tag{2.18}$$

Proof. The proof of Equation 2.18 is by structural induction over the rules T1 to T10 in Table **??**. This is a sound approach because these rules yield all mCRL2 expressions that can result from a translation by T_V. For each translation rule, all representative derivations (see Definition 4.9) for expressions of the form $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{Q}$ are listed, and for each possible derivation Lemma 4.7 is proven.
Induction hypothesis: For all recursions $\mathrm{T}_W(\xi, \mathrm{p})$ that are part of a translation rule defining T_V, it holds that

$$\forall a, \mathrm{Q}\ .\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{Q} \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\smile}, \mathrm{q}, \sigma\ .\ a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q} \wedge \xi, \mathrm{p} \xrightarrow{A_2} \xi[\sigma], \mathrm{q} \wedge \mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{q})$$

Base cases: Show for translation rules T1 to T7 and T10 that for all a and Q such that $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{Q}$ there is some $(A_1, A_2) \in \mathcal{A}^{\smile}\ .\ a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$ and some σ such that $\xi, \mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{q}$ can be derived for all $a' \in A_2$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{q})$.

Example: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \mathbf{broadcast}(ms).\mathrm{p}) = \textstyle\sum_{\mathtt{D}:\mathrm{Set}(\mathrm{IP})} \mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{D} \notin \mathrm{VARS}(\mathrm{T}_\xi(ms)) \cup \mathrm{VARS}(\mathrm{T}_V(\xi, \mathrm{p}))\quad \mathrm{T1}$$

Consider the following derivation for expressions of the form $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{[\![\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms))]\!]} \checkmark}\ \textsc{Axiom}}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \checkmark}\ \text{Definition 4.10}}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{Seq}\ 1}{\left(\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})\right)[\mathtt{D} := \hat{\mathtt{D}}] \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \text{Substitution}}{\textstyle\sum_{\mathtt{D}:\mathrm{Set}(\mathrm{IP})} \mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{Sum}\ 2}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \mathrm{T1}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})}\ \text{Define}\ \sigma := \emptyset$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that

$a \in \{ \textbf{cast}([\![\mathcal{U}_{\text{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!]) \mid \hat{\mathtt{D}} : \text{Set(IP)} \}$ and $\mathrm{Q} = \mathrm{T}_{\text{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$. There is exactly one pair $(A_1, A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\text{DOM}(\xi)}(\xi, \textbf{broadcast}(ms).\mathrm{p}) \xrightarrow{A_1} \mathrm{T}_{\text{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$:

$$(\{ \textbf{cast}([\![\mathcal{U}_{\text{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!]) \mid \hat{\mathtt{D}} : \text{Set(IP)} \}, \{ \textbf{broadcast}(\xi(ms)) \})$$

This pair satisfies the condition that $\xi, \textbf{broadcast}(ms).\mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule BROADCAST (T1)), which is sufficient to prove this particular base case.

Induction step: Given only its side conditions and the induction hypothesis, show for translation rules T8 and T9 that for all a and Q such that $\mathrm{T}_{\text{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{Q}$ there is some $(A_1, A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\text{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$ and some σ such that $\mathrm{p} \xrightarrow{a'} \mathrm{q}$ can be derived for all $a' \in A_2$ and $\mathrm{Q} = \mathrm{T}_{\text{DOM}(\xi[\sigma])}(\text{DOM}(\xi[\sigma]), \mathrm{q})$.

Example: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) = \mathrm{X}(\mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}_\xi(exp_n)) \quad \text{T8}$$
$$\text{where } \mathrm{T}(\mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}) = \mathrm{X}(\mathrm{T}(\mathtt{var}_1) : \text{sort}(\mathrm{T}_\xi(exp_1)), \cdots, \mathrm{T}(\mathtt{var}_n) : \text{sort}(\mathrm{T}_\xi(exp_n))) \stackrel{\text{def}}{=} \mathrm{T}_{\{\mathtt{var}_1, \cdots, \mathtt{var}_n\}}(\emptyset, \mathrm{p})$$

The induction hypothesis states that recursions T_V occurring in $\mathrm{X}(\mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}_\xi(exp_n))$ are related. In particular, the induction hypothesis is used here to relate arbitrary instantiations of process calls:

$$\begin{aligned}
\forall a, \mathrm{Q} \,.\, & \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\}}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p}) \xrightarrow{a} \mathrm{Q} \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^\smile, \mathrm{q}, \sigma \,.\, a \in A_1 \\
& \wedge \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\}}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p}) \xrightarrow{A_1} \mathrm{Q} \\
& \wedge \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p} \xrightarrow{A_2} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q} \\
& \wedge \mathrm{Q} = \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\} \cup \text{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})
\end{aligned}$$

The following derivation in AWN can be based on the third line of the instantiated induction hypothesis:

$$\dfrac{\dfrac{}{\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p} \xrightarrow{a'} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q}}\,\text{Induction hypothesis} \qquad \text{Definition of X}}{\xi, \mathrm{X}(exp_1, \cdots, exp_n) \xrightarrow{a'} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q}}\,\text{RECURSION (T1)}$$

This derivation holds for all $a' \in A_2$. Similarly, in mCRL2, it happens to be the case that

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\}}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p}) \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\} \cup \text{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\,\text{Induction hypothesis}}{\mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\}}(\emptyset, \mathrm{p})[\mathrm{T}(\mathtt{d}_1) := t_{\mathrm{U}(\xi(exp_1))}, \cdots, \mathrm{T}(\mathtt{d}_n) := t_{\mathrm{U}(\xi(exp_n))}] \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\} \cup \text{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\,\text{Lemma 4.3}}{\mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\}}(\emptyset, \mathrm{p})[\mathrm{T}(\mathtt{d}_1) := \mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}(\mathtt{d}_n) := \mathrm{T}_\xi(exp_n)] \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\} \cup \text{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\,\text{Lemma 4.1}}{\mathrm{X}(\mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}_\xi(exp_n)) \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\} \cup \text{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\,\text{Definition of X} \quad \text{RECURSION 2}}{\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\} \cup \text{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\,\text{T8}$$

for all $a \in A_1$. This derivation is a representative derivation without alternatives (see Definition 4.9). Because the induction hypothesis is used to express the relationship between *arbitrary* instantiations of process calls, the first line of this derivation plus the inference rules of mCRL2 must be sufficient to generate all possible behavior of an mCRL2 process call. The actions available to $\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n))$ in mCRL2 can therefore clearly be mimicked by AWN, and so both derivations can be combined into an new equation matching the induction hypothesis:

$$\begin{aligned}
\forall a, \mathrm{Q} \,.\, & \mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{a} \mathrm{Q} \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^\smile, \mathrm{q}, \sigma \,.\, a \in A_1 \\
& \wedge \mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{A_1} \mathrm{Q} \\
& \wedge \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p} \xrightarrow{A_2} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q} \\
& \wedge \mathrm{Q} = \mathrm{T}_{\{\mathtt{d}_1, \cdots, \mathtt{d}_n\} \cup \text{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})
\end{aligned}$$

This confirms the induction step of the proof of Lemma 4.7.

Given the proofs for translation rules T1 to T10 (see Appendix B), Equation 2.18 holds. □

Lemma 4.8 Let $\widetilde{\mathrm{T}}^{\smile}$ be the converse of translation relation $\widetilde{\mathrm{T}}$ from Table **??** and let $\mathcal{A}^{\smile}$ be the converse of relation $\mathcal{A}$ from Table 2.6. Then $\widetilde{\mathrm{T}}^{\smile}$ is a strong $\mathcal{A}^{\smile}$-warped simulation of mCRL2 expressions $\mathrm{T}(\mathrm{P})$ by AWN expressions P for all AWN expressions P.

Proof. Using Lemma 3.1, for Definition 4.5 to apply it is sufficient to prove that

$$\forall \mathrm{P}, a, \mathrm{Q}' \,.\, \Big(\mathrm{T}(\mathrm{P}) \xrightarrow{a} \mathrm{Q}' \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\smile}, \mathrm{Q} \,.\, a \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{Q}' \wedge \mathrm{P} \xrightarrow{A_2} \mathrm{Q} \wedge \mathrm{Q}' = \mathrm{T}(\mathrm{Q}) \Big) \tag{2.19}$$

The proof of Equation 2.19 is provided by structural induction over the translation rules of mCRL2, which is possible because these yield all possible mCRL2 expressions that can result from a translation by T.
Induction hypothesis: For all recursions $\mathrm{T}(\mathrm{P})$ that are part of a translation rule defining T, it holds that

$$\forall a, \mathrm{Q}' \,.\, \Big(\mathrm{T}(\mathrm{P}) \xrightarrow{a} \mathrm{Q}' \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\smile}, \mathrm{Q} \,.\, a \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{Q}' \wedge \mathrm{P} \xrightarrow{A_2} \mathrm{Q} \wedge \mathrm{Q}' = \mathrm{T}(\mathrm{Q}) \Big)$$

Base cases: Show for translation rules T11 and T12 that for all a and Q' such that $\mathrm{T}(\mathrm{P}) \xrightarrow{a} \mathrm{Q}'$ there is some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{Q}'$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{Q}$ can be derived for all $a' \in A_2$ and $\mathrm{Q}' = \mathrm{T}(\mathrm{Q})$.

Example: The translation function T is partially defined by translation rule T12:

$$\mathrm{T}(\xi, \mathrm{p}) = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})$$

Suppose that $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{Q}$. Then, according to Lemma 4.7, there exists some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$ such that $\xi, \mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}'$ for all $a' \in A_2$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p}')$.
Define $\zeta = \xi[\sigma]$. Q can then be rewritten to $\mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta, \mathrm{p}')$ which, as stated by rule T12, equals $\mathrm{T}(\zeta, \mathrm{p}')$. As a result, there must exist some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$ such that $\xi, \mathrm{p} \xrightarrow{a'} \zeta, \mathrm{p}'$ for all $a' \in A_2$ and $\mathrm{Q} = \mathrm{T}(\zeta, \mathrm{p}')$, proving this particular base case.

Induction step: Given only their side conditions and the induction hypothesis, show for translation rules T13 to T16 that for all a and Q' such that $\mathrm{T}(\mathrm{P}) \xrightarrow{a} \mathrm{Q}'$ there is some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{Q}'$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{Q}$ can be derived for all $a' \in A_2$ and $\mathrm{Q}' = \mathrm{T}(\mathrm{Q})$.

Example: The translation function T is partially defined by translation rule

$$\begin{aligned}
&\mathrm{T}([\,\mathrm{M}\,]) = \nabla_V \rho_{\{\mathbf{starcast} \to \mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \quad \text{T16} \\
&\text{where } V = \{\mathbf{t}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\} \\
&\text{where } C = \{\overline{\mathbf{newpkt}} | \mathbf{arrive} \to \mathbf{newpkt}\} \\
&\text{where } \mathrm{H} = \textstyle\sum_{\mathrm{ip:IP,data:DATA,dest:IP}} \overline{\mathbf{newpkt}}(\{\mathtt{ip}\}, \{\mathtt{ip}\}, \mathtt{newpkt}(\mathtt{data}, \mathtt{dest})).\mathrm{H}
\end{aligned}$$

Consider two pairs from Table 2.6:

$$(\{\, H \neg K : \mathbf{arrive}(m) \,\}, \left\{ \mathbf{arrive}([\![\hat{\mathtt{D}}]\!], [\![\hat{\mathtt{D}}\mathtt{'}]\!], \mathrm{U}(m)) \;\middle|\; \begin{smallmatrix} \hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)} \\ \hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}} \\ H \subseteq \hat{\mathtt{D}}\mathtt{'} \\ K \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset \end{smallmatrix} \right\}),$$

$$(\{\, ip : \mathbf{newpkt}(d, dip) \,\}, \{\, \mathbf{newpkt}(\{\mathrm{U}(ip)\}, \{\mathrm{U}(ip)\}, \mathtt{newpkt}(\mathrm{U}(d), \mathrm{U}(dip))) \,\})$$

Let A_1 and N_1 be defined as the second members of these pairs; that is, let

$$A_1 \stackrel{\text{def}}{=} \left\{ \mathbf{arrive}([\![\hat{\mathtt{D}}]\!], [\![\hat{\mathtt{D}}\mathtt{'}]\!], \mathrm{U}(m)) \;\middle|\; \begin{smallmatrix} \hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)} \\ \hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}} \\ H \subseteq \hat{\mathtt{D}}\mathtt{'} \\ K \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset \end{smallmatrix} \right\}$$

$$N_1 \stackrel{\text{def}}{=} \{\, \mathbf{newpkt}(\{\mathrm{U}(ip)\}, \{\mathrm{U}(ip)\}, \mathtt{newpkt}(\mathrm{U}(d), \mathrm{U}(dip))) \,\}$$

and let

$$\overline{N}_1 \stackrel{\text{def}}{=} \left\{ \overline{\mathbf{newpkt}}(\{\mathrm{U}(ip)\}, \{\mathrm{U}(ip)\}, \mathtt{newpkt}(\mathrm{U}(d), \mathrm{U}(dip))) \right\}$$

for some d : DATA, ip : IP, H, K : Set(IP), and m : MSG.
The following cases are distinguished:

1: $\mathrm{T}(\mathrm{M})$ does a transition $a \in A_1$ and H does a transition $b \in \overline{N}_1$.
2: $\mathrm{T}(\mathrm{M})$ does a transition $a \notin A_1$ and H does a transition $b \in \overline{N}_1$.
3: $\mathrm{T}(\mathrm{M})$ does nothing and H does a transition $b \in \overline{N}_1$.
4: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} =$ **starcast** and H does nothing.
5: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} \in \{$**t**, **newpkt**, **deliver**, **connect**, **disconnect**$\}$ and H does nothing.
6: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} \notin \{$**t**, **starcast**, **newpkt**, **deliver**, **connect**, **disconnect**$\}$ and H does nothing.

Note that these cases cover all combinations of behavior of $\mathrm{T}(\mathrm{M})$ and H (H can only do $\overline{\textbf{newpkt}}$ actions; see the first part of the derivation in the Lemma 4.6-proof for Newpkt (T4), which is a representative derivation without alternatives).
The proof is provided below for each of the cases:

1: $\mathrm{T}(\mathrm{M})$ does a transition $a \in A_1$ and H does a transition $b \in \overline{N}_1$. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from A_1, it can first be concluded that

$$\mathrm{T}(\mathrm{M}) \xrightarrow{L_1} \mathrm{T}(\mathrm{M}') \wedge \mathrm{M} \xrightarrow{H\neg K:\textbf{arrive}(m)} \mathrm{M}'$$

The mCRL2 derivation in the Lemma 4.6-proof for Newpkt (T4) shows how the first conjunct is sufficient to prove that $\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{L_1} \mathrm{T}([\ \mathrm{M}'\ ])$. Under the constraints of this case, the derivation is a representative derivation without alternatives (see Definition 4.9) and so all related behavior of $\mathrm{T}([\ \mathrm{M}\ ])$ is covered.
On the AWN side, the second conjunct can be used as premise in

$$\frac{\mathrm{M} \xrightarrow{\{ip\}\neg K:\textbf{arrive}(\mathtt{newpkt}(d,dip))} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{ip:\textbf{newpkt}(d,dip)} [\ \mathrm{M}'\ ]}\ \textsc{Newpkt (T4)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{A_1} \mathrm{T}([\ \mathrm{M}'\ ]) \wedge [\ \mathrm{M}\ ] \xrightarrow{A_2} [\ \mathrm{M}'\ ]$, and the converse of Pair 2.16 satisfies this requirement.
2: $\mathrm{T}(\mathrm{M})$ does a transition $a \notin A_1$ and H does a transition $b \in \overline{N}_1$.
This situation fails to generate behavior for $\mathrm{T}([\ \mathrm{M}\ ])$ because no derivation similar to the one in the Lemma 4.6-proof for Newpkt (T4) is possible:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{T}(\mathrm{M}) \xrightarrow{a} \mathrm{T}(\mathrm{M}') \qquad \mathrm{H} \xrightarrow{\overline{\textbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{H}}{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H} \xrightarrow{a\mid\overline{\textbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H}}\ \textsc{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\gamma_C(a\mid\overline{\textbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip))))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{a\mid\overline{\textbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\textbf{starcast}\to\textbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\{\textbf{starcast}\to\textbf{t}\}\bullet a\mid\overline{\textbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\textbf{starcast}\to\textbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \textsc{Rename 2}}{\rho_{\{\textbf{starcast}\to\textbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{a\mid\overline{\textbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\textbf{starcast}\to\textbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \text{Apply } \{\textbf{starcast}\to\textbf{t}\}\bullet$$

The Allow 2 operator cannot be applied next because $\underline{a}|\overline{\textbf{newpkt}} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}([\ \mathrm{M}\ ])$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.
3: $\mathrm{T}(\mathrm{M})$ does nothing and H does a transition $\overline{\textbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip))) \in \overline{N}_1$.
This situation fails to generate behavior for $\mathrm{T}([\ \mathrm{M}\ ])$ because no derivation similar to the one in the Lemma 4.6-proof for Newpkt (T4) is possible:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{H}}{\mathrm{T(M)} \parallel \mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{T(M)} \parallel \mathrm{H}}\ \textsc{Par 5}}{\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{\gamma_C(\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip))))} \Gamma_C(\mathrm{T(M)} \parallel \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \Gamma_C(\mathrm{T(M)} \parallel \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H})}\ \textsc{Rename 2}}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H})}\ \text{Apply } \{\mathbf{starcast}\to\mathbf{t}\}\bullet$$

The Allow 2 operator cannot be applied next because $\overline{\mathbf{newpkt}} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}([\ \mathrm{M}\ ])$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

4: $\mathrm{T(M)}$ does a transition a where $\underline{a} = \mathbf{starcast}$ and H does nothing.
Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition label **starcast**, it can first be concluded that

$$\mathrm{T(M)} \xrightarrow{\mathbf{starcast}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T(M')} \wedge \mathrm{M} \xrightarrow{R\text{:}*\mathbf{cast}(m)} \mathrm{M'}$$

for all D, R : Set(IP) and m : MSG where $R \subseteq D$.
The mCRL2 derivation in the Lemma 4.6-proof for Cast (T4-4) shows how the first conjunct is sufficient to prove that $\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T}([\ \mathrm{M'}\ ])$. Under the constraints of this case, the derivation is a representative derivation without alternatives (see Definition 4.9) and so all related behavior of $\mathrm{T}([\ \mathrm{M}\ ])$ is covered.
On the AWN side, the second conjunct can be used as premise in

$$\dfrac{\mathrm{M} \xrightarrow{R:*\mathbf{cast}(m)} \mathrm{M'}}{[\ \mathrm{M}\ ] \xrightarrow{\tau} [\ \mathrm{M'}\ ]}\ \textsc{Cast (T4-4)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{A_1} \mathrm{T}([\ \mathrm{M'}\ ]) \wedge [\ \mathrm{M}\ ] \xrightarrow{A_2} [\ \mathrm{M'}\ ]$, and the converse of Pair 2.11 satisfies this requirement.

5: $\mathrm{T(M)}$ does a transition a where $\underline{a} \in \{\mathbf{t}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}$ and H does nothing.
The induction hypothesis states that

$$\exists (A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T(P)} \xrightarrow{A_1} \mathrm{T(P')} \wedge \mathrm{P} \xrightarrow{A_2} \mathrm{P'}$$

Define $\mathcal{A}'$ as a subset of $\mathcal{A}^{\smile}$ that contains the possible values of $(A_1, A_2) \in \mathcal{A}^{\smile}$ where A_1 are actions that $\mathrm{T(P} \lll \mathrm{Q)}$ can perform in mCRL2 and where A_2 are the actions that AWN should use to mimic the actions in A_1 in order for this case to be proven:

$$\mathcal{A}' \stackrel{\text{def}}{=} \left\{ (A_1, A_2) \;\middle|\; (A_1, A_2) \in \mathcal{A}^{\smile}, \quad \mathrm{T(P} \lll \mathrm{Q)} \xrightarrow{A_1} \mathrm{T(P'} \lll \mathrm{Q)} \right\}$$

The following derivation is possible in mCRL2 for all $a \in A_1$ such that $(A_1, A_2) \in \mathcal{A}'$:

$$a \underline{\in} V \cup \{\tau\}\ \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T(M)} \xrightarrow{a} \mathrm{T(M')}}\ \text{Induction hypothesis}}{\mathrm{T(M)} \parallel \mathrm{H} \xrightarrow{a} \mathrm{T(M')} \parallel \mathrm{H}}\ \textsc{Par 2}}{\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{\gamma_C(a)} \Gamma_C(\mathrm{T(M')} \parallel \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{a} \Gamma_C(\mathrm{T(M')} \parallel \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet a} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \parallel \mathrm{H})}\ \textsc{Rename 2}}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{a} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \parallel \mathrm{H})}\ \text{Apply } \{\mathbf{starcast}\to\mathbf{t}\}\bullet}{\nabla_V \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \parallel \mathrm{H}) \xrightarrow{a} \nabla_V \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \parallel \mathrm{H})}\ \textsc{Allow 2}}{\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{a} \mathrm{T}([\ \mathrm{M'}\ ])}\ \textsc{T16}$$

The derivation above is valid only for $a \in A_1$ such that $\underline{a} \in V \cup \{\tau\}$. Under the constraints of this case, the derivation is also a representative derivation without alternatives (see Definition 4.9). Finally it must be observed that it is impossible for $\mathrm{T(M)}$ to produce a **newpkt** action independent of H, meaning that $\underline{a} \neq \mathbf{newpkt}$. Consequently, $\mathcal{A}'$ is as follows:

$$
\begin{aligned}
\{ \quad & \\
& (\{\ \mathbf{t}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))\ \}, \{\ \tau\ \}), \\
& (\{\ \mathbf{deliver}(\mathrm{U}(ip), \mathrm{U}(d))\ \}, \{\ ip\!:\!\mathbf{deliver}(d)\ \}), \\
& (\{\ \mathbf{connect}(\mathrm{U}(ip'), \mathrm{U}(ip''))\ \}, \{\ \mathbf{connect}(ip', ip'')\ \}), \\
& (\{\ \mathbf{disconnect}(\mathrm{U}(ip'), \mathrm{U}(ip''))\ \}, \{\ \mathbf{disconnect}(ip', ip'')\ \}) \\
& \\
| \quad & m : \mathrm{MSG};\quad ip, ip', ip'' : \mathrm{IP};\quad d : \mathrm{DATA};\quad dests, R, D : \mathrm{Set(IP)};\quad R \subseteq D \quad \}
\end{aligned}
$$

For all $(A_1, A_2) \in \mathcal{A}'$ the actions A_1 in mCRL2 can be mimicked by the actions A_2 in AWN by means of the inference rules

$$\frac{\mathrm{M} \xrightarrow{\tau} \mathrm{M}'}{[\,\mathrm{M}\,] \xrightarrow{\tau} [\,\mathrm{M}'\,]}\ \textsc{Internal} \text{ (T4-3)}$$

$$\frac{\mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}'}{[\,\mathrm{M}\,] \xrightarrow{ip:\mathbf{deliver}(d)} [\,\mathrm{M}'\,]}\ \textsc{Deliver} \text{ (T4-3)}$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{connect}(ip,ip')} \mathrm{M}'}{[\,\mathrm{M}\,] \xrightarrow{\mathbf{connect}(ip,ip')} [\,\mathrm{M}'\,]}\ \textsc{Connect} \text{ (T4-2)}$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{disconnect}(ip,ip')} \mathrm{M}'}{[\,\mathrm{M}\,] \xrightarrow{\mathbf{disconnect}(ip,ip')} [\,\mathrm{M}'\,]}\ \textsc{Disconnect} \text{ (T4-2)}$$

respectively. Therefore the induction hypothesis holds for this case.

6: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} \notin \{\mathbf{t}, \mathbf{starcast}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}$ and H does nothing. This situation fails to generate behavior for $\mathrm{T}([\,\mathrm{M}\,])$ because no derivation similar to the one in the Lemma 4.6-proof for Newpkt (T4) is possible:

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{T}(\mathrm{M}) \xrightarrow{a} \mathrm{T}(\mathrm{M}')}{\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H} \xrightarrow{a} \mathrm{T}(\mathrm{M}') \,||\, \mathrm{H}}\ \textsc{Par 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\gamma_C(a)} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{a} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet a} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\ \textsc{Rename 2}$$

The $\textsc{Allow 2}$ operator cannot be applied next because $\underline{a} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}([\,\mathrm{M}\,])$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

Similar proofs must be done for all other translation rules defining T. These proofs can be found in Appendix C. Given all of the proofs, Equation 2.19 holds. □

Theorem 4.9 Take translation relation $\widetilde{\mathrm{T}}$ from Table **??** and action relation $\mathcal{A}$ from Table 2.6. Then $\widetilde{\mathrm{T}}$ is a strong $\mathcal{A}$-warped bisimulation between AWN expressions P and mCRL2 expressions $\mathrm{T}(\mathrm{P})$ for all AWN expressions P.

Proof. $\widetilde{\mathrm{T}}$ is a strong $\mathcal{A}$-warped bisimulation of P by $\mathrm{T}(\mathrm{P})$ for all P if and only if Definition 4.6 applies. This definition consists of two conditions, the first of which is established by the proof for Lemma 4.6 and the second of which is established by the proof for Lemma 4.8. □

Theorem 4.10 Let

$$\widetilde{\mathrm{T}}_\tau \stackrel{\text{def}}{=} \{\ ([\,\mathrm{M}\,], \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]))\ \big|\ [\,\mathrm{M}\,] \text{ is an AWN network expression}\ \} \tag{2.20}$$

Then there exists a bijective function $\mathfrak{f} : \mathrm{Act}_{\mathrm{AWN}} \to \mathrm{Act}_{\mathrm{mCRL2}}$ so that $\widetilde{\mathrm{T}}_\tau$ is a strong bisimulation between AWN expressions $[\,\mathrm{M}\,]$ and mCRL2 expressions $\mathrm{T}([\,\mathrm{M}\,])$ modulo renaming by $\mathfrak{f}$ for all AWN expressions $[\,\mathrm{M}\,]$.

Proof. The inference rules of AWN (see Tables 2.1 and 2.2) clearly show that

$$\forall \mathrm{M}, \mathrm{Q}, a \,.\, [\,\mathrm{M}\,] \xrightarrow{a} \mathrm{Q} \quad \Rightarrow \quad \exists \mathrm{M}' \,.\, \mathrm{Q} = [\,\mathrm{M}'\,]$$

which means that Equation 2.17 implies that

$$\forall \mathrm{M}, \mathrm{M}', a \,.\, \Big([\,\mathrm{M}\,] \xrightarrow{a} [\,\mathrm{M}'\,] \Rightarrow \exists (A_1, A_2) \in \mathcal{A} \,.\, a \in A_1 \wedge [\,\mathrm{M}\,] \xrightarrow{A_1} [\,\mathrm{M}'\,] \wedge \mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{A_2} \mathrm{T}([\,\mathrm{M}'\,]) \Big) \tag{2.21}$$

Similarly, the Lemma 4.8-proof for Translation rule T16 shows that

$$\forall \mathrm{M}, \mathrm{Q}, a \,.\, [\,\mathrm{T}(\mathrm{M})\,] \xrightarrow{a} \mathrm{Q} \quad \Rightarrow \quad \exists \mathrm{M}' \,.\, \mathrm{Q} = \mathrm{T}([\,\mathrm{M}'\,])$$

which means that Equation 2.19 implies that

$$\forall \mathrm{M}, \mathrm{M}', a' \,.\, \Big(\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{a'} \mathrm{T}([\,\mathrm{M}'\,]) \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a' \in A_1 \wedge \mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{A_1} \mathrm{T}([\,\mathrm{M}'\,]) \wedge [\,\mathrm{M}\,] \xrightarrow{A_2} [\,\mathrm{M}'\,] \Big) \tag{2.22}$$

Equations 2.21 and 2.22 only require a subset $\mathcal{A}'$ of $\mathcal{A}$ in order to be valid, namely the subset containing the pairs in $\mathcal{A}$ with the actions that $[\,\mathrm{M}\,]$ and $\mathrm{T}([\,\mathrm{M}\,])$ can actually perform. To determine the contents of $\mathcal{A}'$, the inference rules of AWN and the Lemma 4.8-proof for Translation rule T16 can be used once again because they list all possible behavior of $[\,\mathrm{M}\,]$ and $\mathrm{T}([\,\mathrm{M}\,])$. This results in the following value of $\mathcal{A}'$:

$$\begin{aligned}
\mathcal{A}' = \{ & \\
& (\{\,\tau\,\}, \{\,\mathbf{t}(\mathsf{U}(D), \mathsf{U}(R), \mathsf{U}(m))\,\}), \\
& (\{\,ip\!:\!\mathbf{deliver}(d)\,\}, \{\,\mathbf{deliver}(\mathsf{U}(ip), \mathsf{U}(d))\,\}), \\
& (\{\,\mathbf{connect}(ip', ip'')\,\}, \{\,\mathbf{connect}(\mathsf{U}(ip'), \mathsf{U}(ip''))\,\}), \\
& (\{\,\mathbf{disconnect}(ip', ip'')\,\}, \{\,\mathbf{disconnect}(\mathsf{U}(ip'), \mathsf{U}(ip''))\,\}), \\
& (\{\,ip\!:\!\mathbf{newpkt}(d, dip)\,\}, \{\,\mathbf{newpkt}(\{\mathsf{U}(ip)\}, \{\mathsf{U}(ip)\}, \mathtt{newpkt}(\mathsf{U}(d), \mathsf{U}(dip)))\,\}) \\
\mid \; & m : \mathrm{MSG};\quad ip, ip', ip'', dip : \mathrm{IP};\quad d : \mathrm{DATA};\quad R, D : \mathrm{Set(IP)};\quad R \subseteq D \quad \}
\end{aligned}$$

Furthermore, the different manifestations of the **t** action are not visible at the network level, and therefore they can be hidden in mCRL2 using a $\tau_{\{\mathbf{t}\}}$ operation:

$$\frac{\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{a'} \mathrm{T}([\,\mathrm{M}'\,])}{\tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{\theta_{\{\mathbf{t}\}}(a')} \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}'\,])}\ \textsc{Hide}\ 2$$

(*Is this step sufficiently straightforward?*)

This changes Equations 2.21 and 2.22 and the value of $\mathcal{A}'$ to the following:

$$\forall \mathrm{M}, \mathrm{M}', a \,.\, \Big([\,\mathrm{M}\,] \xrightarrow{a} [\,\mathrm{M}'\,] \Rightarrow \exists (A_1, A_2) \in \mathcal{A}'_\tau \,.\, a \in A_1 \wedge [\,\mathrm{M}\,] \xrightarrow{A_1} [\,\mathrm{M}'\,] \wedge \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{A_2} \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}'\,]) \Big) \tag{2.23}$$

$$\forall \mathrm{M}, \mathrm{M}', a' \,.\, \Big(\tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{a'} \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}'\,]) \Rightarrow \exists (A_1, A_2) \in \mathcal{A}'^{\smile}_\tau \,.\, a' \in A_1\tau_{\{\mathbf{t}\}} \wedge \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{A_1} \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}'\,]) \wedge [\,\mathrm{M}\,] \xrightarrow{A_2} [\,\mathrm{M}'\,] \Big) \tag{2.24}$$

where $\mathcal{A}'_\tau \stackrel{\text{def}}{=} \{$

$$\begin{aligned}
& (\{\,\tau\,\}, \{\,\tau\,\}), \\
& (\{\,ip\!:\!\mathbf{deliver}(d)\,\}, \{\,\mathbf{deliver}(\mathsf{U}(ip), \mathsf{U}(d))\,\}), \\
& (\{\,\mathbf{connect}(ip', ip'')\,\}, \{\,\mathbf{connect}(\mathsf{U}(ip'), \mathsf{U}(ip''))\,\}), \\
& (\{\,\mathbf{disconnect}(ip', ip'')\,\}, \{\,\mathbf{disconnect}(\mathsf{U}(ip'), \mathsf{U}(ip''))\,\}), \\
& (\{\,ip\!:\!\mathbf{newpkt}(d, dip)\,\}, \{\,\mathbf{newpkt}(\{\mathsf{U}(ip)\}, \{\mathsf{U}(ip)\}, \mathtt{newpkt}(\mathsf{U}(d), \mathsf{U}(dip)))\,\})
\end{aligned}$$

$$\mid \; ip, ip', ip'', dip : \mathrm{IP};\quad d : \mathrm{DATA} \quad \}$$

Note that $\mathcal{A}'_\tau$ can be replaced by the bijective function $\mathfrak{f}$:

$$\forall \mathrm{M}, \mathrm{M}', a \,.\, \Big([\,\mathrm{M}\,] \xrightarrow{a} [\,\mathrm{M}'\,] \Rightarrow \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{\mathfrak{f}(a)} \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}'\,]) \Big) \tag{2.25}$$

$$\forall \mathrm{M}, \mathrm{M}', a' \,.\, \Big(\tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{a'} \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}'\,]) \Rightarrow [\,\mathrm{M}\,] \xrightarrow{\mathfrak{f}^{-1}(a')} [\,\mathrm{M}'\,] \Big) \tag{2.26}$$

$$\text{where } \mathfrak{f}(a) = \begin{cases} \tau & \text{if } a = \tau \\ \mathbf{deliver}(\mathsf{U}(ip), \mathsf{U}(d)) & \text{if } a = ip\!:\!\mathbf{deliver}(d) \\ \mathbf{connect}(\mathsf{U}(ip'), \mathsf{U}(ip'')) & \text{if } a = \mathbf{connect}(ip', ip'') \\ \mathbf{disconnect}(\mathsf{U}(ip'), \mathsf{U}(ip'')) & \text{if } a = \mathbf{disconnect}(ip', ip'') \\ \mathbf{newpkt}(\{\mathsf{U}(ip)\}, \{\mathsf{U}(ip)\}, \mathtt{newpkt}(\mathsf{U}(d), \mathsf{U}(dip))) & \text{if } a = ip\!:\!\mathbf{newpkt}(d, dip) \end{cases}$$

from which can be concluded immediately that

$$\{\ ([\,\mathrm{M}\,], \tau_{\{\mathbf{t}\}}\mathrm{T}([\,\mathrm{M}\,]))\ \big|\ [\,\mathrm{M}\,] \text{ is an AWN network expression}\ \}$$

is a strong bisimulation modulo renaming by $\mathfrak{f}$. □

Theorem 4.11 $\mathcal{F}$ does not matter with regard to strong bisimulation. Why? Because. **TODO**

Proof. **TODO** □

Part 3

Implementation

1 Design principles

As stated in the Introduction, one of the ambitions for AWN is to automatically model and verify AWN specifications with one or more existing model checking toolsets. Supporting multiple toolsets means that building a translation framework rather than a situational solution will yield the most long-term benefits. For the project, such a framework has been developed based on the principle of *model-driven engineering* [15] [20] or *MDE*.
The MDE principle – an overview of which is depicted in Figure 3.1 – can be summarized with the phrase "Everything is a model", which is remarkably similar to the "Everything is an object" phrase that was used in the 1980s to describe the essence of object-oriented techniques [2]. Models express structure, behavior, and other properties in a relevant domain. The permitted *syntax* of a model is expressed by a metamodel, which in turn conforms to the syntax expressed by a metametamodel (the syntax of a metametamodel can be expressed by the metametamodel itself). Sharing a metametamodel, such as the one introduced in 2000 by the Object Management Group [21], makes metamodels compatible.
Another important component of model-driven engineering is the application of *(model) transformations*. These transformation are defined in terms of the metamodels of the input and output model. Model transformations are typically defined in a domain-specific language (DSL) for transformations and executed on a *transformation engine*.

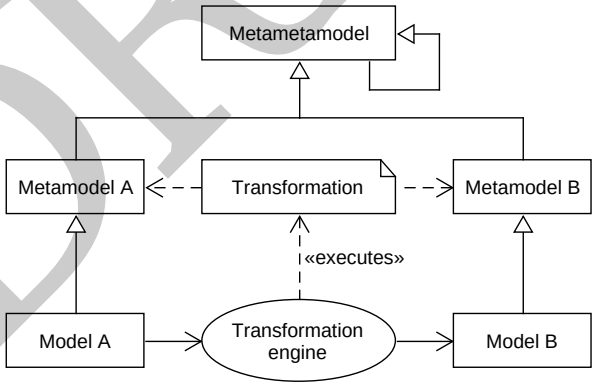

Figure 3.1: *How to transform some model A to some model B using the MDE principle.*

MDE also allows tools to be developed that can handle input that is not model-specific or even metamodel-specific. Instead, the input must consist of a model conforming to a metamodel and a definition of that metamodel (conforming to a predefined metametamodel). This has given rise to frameworks such as Xtext (used for this project), which can generate compilers and editors from given metamodels.

The implementation of the AWN-to-mCRL2 translation has been built using existing MDE frameworks and techniques. This should bring the following advantages:

- The standard is set high for the degree at which the different aspects of an implementation are separated, similar to *aspect-oriented techniques* [16]. This improves code maintainability.
- It is easy to benefit from existing frameworks such as Xtext, which have been developed with the compatibility of metametamodels in mind.
- The implementation can be more easily combined with other MDE projects (assuming a shared metametamodel), reducing efforts to reuse code.

2 Implementation overview

Unsurprisingly, the implementation of the AWN-to-mCRL2 translation is given an AWN specification and, if the AWN specification is valid, returns an mCRL2 specification. The conversion occurs via a chain of transformations, similar to a pipeline (see Figure 3.2).
The following four transformations are executed in order. First, the AWN specification is parsed and validated, resulting in a 'raw' AWN model (similar to an abstract syntax tree) if successful. This model is then reorganized and decorated with more information, which gives a more usable AWN model. The third transformation constructs an mCRL2 model based on the ingoing AWN model, and this transformation therefore represents the essence of the AWN-to-mCRL2 translation. The fourth step involves exporting the mCRL2 model as one or more text files.

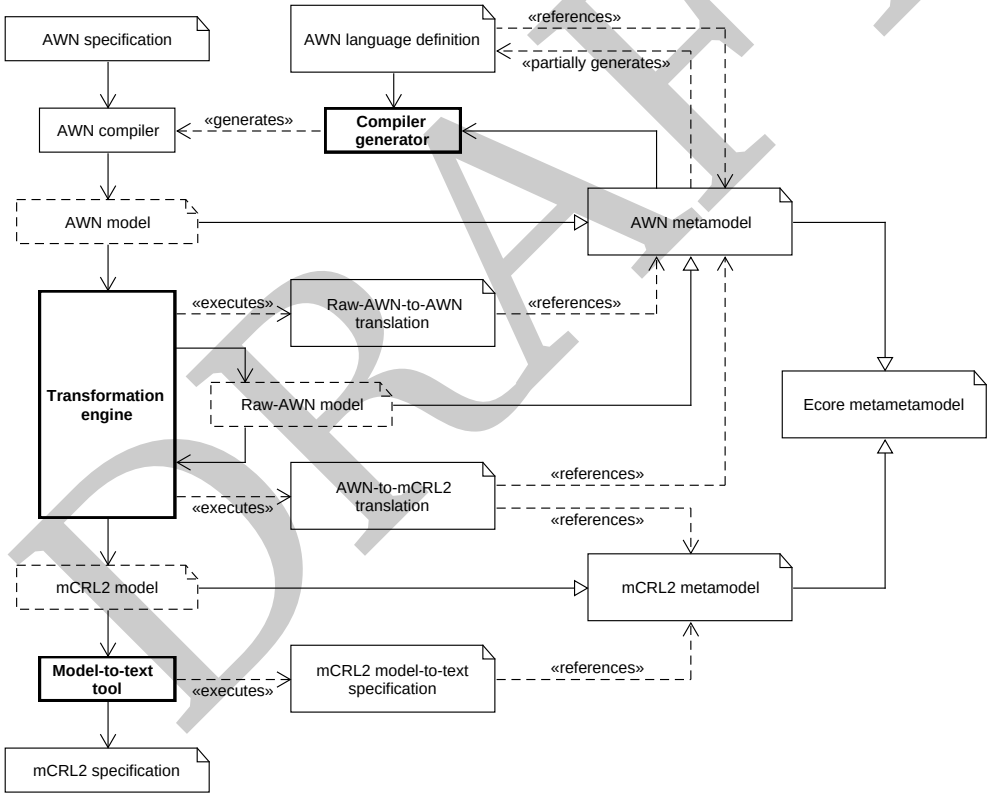

Figure 3.2: *Chain of transformations used by the implementation of the AWN-to-mCRL2 translation.*

Other translations (a translation from AWN to UPPAAL, for example) may require additional intermediate steps or not intermediate steps at all. In order to facilitate a multi-target translation framework, a domain-specific language called TCL (Transformation Chain Language) has been developed in which transformation chains are specified. TCL specification consist of a single plain-text file with a `.chain` extension. Typically, each type of translation has one corresponding `.chain` file.

3 AWN-to-mCRL2 transformation chain

This section will discuss the steps that form the AWN-to-mCRL2 transformation chain in greater detail. Figure 3.3 gives an even more simplified overview of this transformation chain than Figure 3.2:

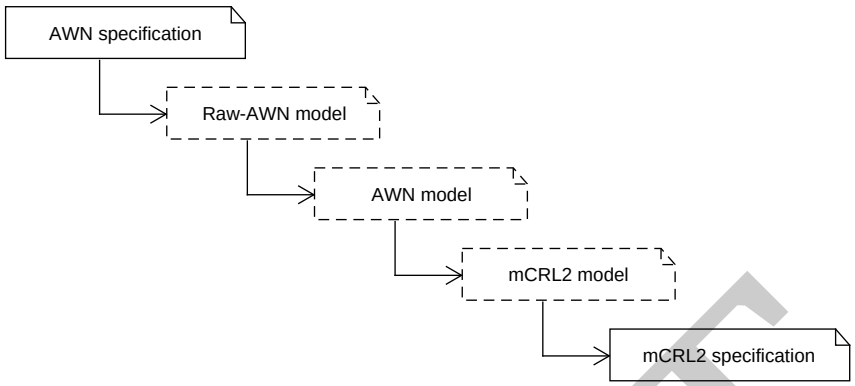

Figure 3.3: *Simplified transformation chain for the AWN-to-mCRL2 translation.*

3.1 Compilation of AWN specifications

The first transformation in the transformation chain is 'simply' a compilation of text: the AWN specification in text form is converted by a lexer to a token stream, which is subsequently used by a parser to construct a 'raw' AWN model instance (as if it were an abstract syntax tree). Finally, the 'raw' AWN model is checked for problems by the validator. The transformation chain is aborted if problems are found.

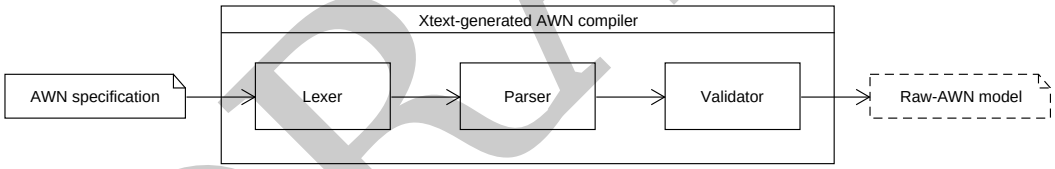

Figure 3.4: *Compiling an AWN specification with Xtext-generated components.*

The lexer and parser mentioned above are all generated from a language specification by Xtext, a framework for developing domain-specific languages. The tokens that the lexer distinguishes are listed in Table **??**. The grammar used by the parser can be found across Section **??**, where the syntax of AWN (as developed in the course of this project) is discussed.
Of the validator only a skeleton can be generated by Xtext because the language specification capable of defining only a small number of model restrictions. The validator has therefore been implemented almost entirely by hand. The implemented validator contains up to one method per type of element of an AWN model, and the body of such a method is used to validate the contents of a particular instance of that element type. Several (Java) helper classes have been written to assist with the validation.
Xtext automatically links the lexer, parser, and validator to the compiler. Besides these bare necessities, Xtext also provides some essential components for an integrated development environment (IDE), such as a scope provider (which dictates which objects are accessible at a certain position in the AWN specification) and a content assistant (which helps the user with modifying code). Several of these components have been implemented in order to make the Xtext-generated IDE for AWN usable.

3.2 Transformation from Raw-AWN to AWN

The 'raw' AWN model produced by the AWN compiler contains more than likely certain shortcomings. For example, the model contains elements of a type called `NamedDefinitionObjectRef`, which could in actuality represent a typecast, a variable reference, a function call, or one of the values of an enumerable type. The AWN compiler is unable to differentiate between these situations until the model is already constructed, and because each of these situations requires a different type of translation, differentiation must occur at some later time.
These sort of tasks are performed as much as possible during the transformation from Raw-AWN to AWN. Such tasks include:

- Introducing actual objects that represent primitive types, and changing references to primitive types to references to those objects.
- Giving each variable a unique name in order to preempt naming conflicts.
- Replacing elements in the model by more specialized elements so that differentiation is easier at later stages of the translation. The prime example of this has already been mentioned at the start of this section, namely `NamedDefinitionObjectRef`.
- Flattening declarations of variables, quantifiers, and parameters.
- Resolving syntactic sugar. Several AWN language constructs can be rewritten to a more generic language construct: an **ifexists** expression is a strengthened version of a **with-init** expression, for example.
- Choosing the actual operator type of an operation based on the type of the operands. The '+' symbol, for example, can be used to add to integers or to concatenate two lists. The compiler always assumes the former, relying on the fact that the operator type will be corrected later.
- Converting the linear process structure used by the 'raw' AWN model to a tree structure.

3.3 Transformation from AWN to mCRL2

This part of the implementation has been, of course, the focus of the entire project: it is the code where AWN is the input and mCRL2 is the output, and where the model conversion actually takes place.
The AWN-to-mCRL2 translation is contained entirely in the file `awn2mcrl2.qvto`. The translation consists of several other tasks besides the application of the translation function defined in Table **??**. These other tasks include translating data expressions, generating data types, and generating mappings and equations in order to manipulate data objects. It should be noted that the translation produces one mCRL2 models for each network topology that has been specified. The workflow that `awn2mcrl2.qvto` follows for each network topology is depicted in Figure **??**.

First, the translation generates declarations of all primitive types. This does not really matter for booleans, integers, and strings, but TODO. The other primitive types are struct types, and declaring them is probably unavoidable. The struct types are not yet given any fields, this occurs later. The translation also generates a declaration of Set(IP) because this type is used a large number of times by action declarations.
Second, the translation declares the actions that are needed. Third, the translation determines which struct fields a struct should have (TODO is there a problem with the code there?). Enumerables are assigned enum values. Function types are constructed even when not explicitly defined (partial functions can be constructed in the AWN code). When all types have been declared, types can start referring to each other.
Then the translation goes through the following, in order:

1. Constants (can add equations).
2. Functions (can add equations).
3. Equations (only if there actually are equations).
4. Sequential processes and parallel processes. The translation rules are applied for this.
5. Network glue process and node glue process, based on the translation rules.
6. Network topology, based on the translation rules.

3.4 mCRL2 to text

The model-to-text part of the translation was implemented with the following features:

- Model-to-text specifications are compiled at runtime. This has the advantage that users can add or modify a translation without having to rebuild the AWN plug-in for Eclipse.
- Names and locations of the output files are specified in the current `.chain` file. This means that all information related to input and output can be found in that `.chain` file.

The model-to-text translation for mCRL2 was defined in a metamodel-independent language developed specifically for this project, namely `TxtGen`. Not making use of an existing tool deviates from the MDE principle to reuse previous work as much as possible; however, available tools such as Xpand and Acceleo required more effort than expected to implement the features mentioned above. In the future, it may be preferable to replace the mCRL2-to-text translation.

4 Unit testing

Not even been implemented...

Conclusions

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Appendix A

Complete proof of Lemma 4.6

1 Base cases

Broadcast (T1): AWN defines the inference rule

$$\frac{}{\xi, \mathbf{broadcast}(ms).\mathrm{p} \xrightarrow{\mathbf{broadcast}(\xi(ms))} \xi, \mathrm{p}}\ \textsc{Broadcast (T1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{broadcast}(\xi(ms)) \in A_1 \wedge \xi, \mathbf{broadcast}(ms).\mathrm{p} \xrightarrow{A_1} \xi, \mathrm{p}$, namely Pair 2.4 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi, \mathrm{p})$ for all $a \in A_2 = \left\{ \mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!]) \mid \hat{\mathtt{D}} : \mathrm{Set(IP)} \right\}$.
In mCRL2, the following derivation can be made for all $\hat{\mathtt{D}}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{[\![\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms))]\!]} \checkmark}\ \textsc{Axiom}}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{\mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \checkmark}\ \text{Definition 4.10}}{\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{Seq 1}}{(\mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}))[\mathtt{D} := \hat{\mathtt{D}}] \xrightarrow{\mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \text{Substitution}}{\sum_{\mathtt{D}:\mathrm{Set(IP)}} \mathbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \textsc{Sum 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\ \text{T1}}{\mathrm{T}(\xi, \mathbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}(\xi, \mathrm{p})}\ \text{T12}$$

for $\hat{\mathtt{D}} : \mathrm{Set(IP)}$ and $\mathtt{D} \notin \textsc{vars}(\mathrm{T}_\xi(ms)) \cup \textsc{vars}(\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}))$. In conclusion, the induction hypothesis holds for this base case.

Groupcast (T1): AWN defines the inference rule

$$\frac{}{\xi, \mathbf{groupcast}(dests, ms).\mathrm{p} \xrightarrow{\mathbf{groupcast}(\xi(dests), \xi(ms))} \xi, \mathrm{p}}\ \textsc{Groupcast (T1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{groupcast}(\xi(dests), \xi(ms)) \in A_1 \wedge \xi, \mathbf{groupcast}(ms).\mathrm{p} \xrightarrow{A_1} \xi, \mathrm{p}$, namely Pair 2.5 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi, \mathbf{groupcast}(ms).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi, \mathrm{p})$ for all $a \in A_2 = \left\{ \mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!]) \mid \hat{\mathtt{D}} : \mathrm{Set(IP)} \right\}$.
In mCRL2, the following derivation can be made for all $\hat{\mathtt{D}}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{cast}(\mathrm{T}_\xi(dests),\hat{\mathtt{D}},\mathrm{T}_\xi(ms)) \xrightarrow{[\![\mathbf{cast}(\mathrm{T}_\xi(dests),\hat{\mathtt{D}},\mathrm{T}_\xi(ms))]\!]} \checkmark}\,\text{Axiom}}{\mathbf{cast}(\mathrm{T}_\xi(dests),\hat{\mathtt{D}},\mathrm{T}_\xi(ms)) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!],[\![\hat{\mathtt{D}}]\!],[\![\mathrm{T}_\xi(ms)]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{cast}(\mathrm{T}_\xi(dests),\hat{\mathtt{D}},\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!],[\![\hat{\mathtt{D}}]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Seq 1}}{\left(\mathbf{cast}(\mathrm{T}_\xi(dests),\hat{\mathtt{D}},\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})\right)[\mathtt{D}:=\hat{\mathtt{D}}] \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!],[\![\hat{\mathtt{D}}]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Substitution}}{\sum_{\mathtt{D}:\mathrm{Set(IP)}} \mathbf{cast}(\mathrm{T}_\xi(dests),\mathtt{D},\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!],[\![\hat{\mathtt{D}}]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Sum 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{groupcast}(dests,ms).\mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!],[\![\hat{\mathtt{D}}]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{T2}}{\mathrm{T}(\xi,\mathbf{groupcast}(dests,ms).\mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!],[\![\hat{\mathtt{D}}]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}(\xi,\mathrm{p})}\,\text{T12}$$

for $\hat{\mathtt{D}} : \mathrm{Set(IP)}$ and $\mathtt{D} \notin \mathrm{VARS}(\mathrm{T}_\xi(dests)) \cup \mathrm{VARS}(\mathrm{T}_\xi(ms)) \cup \mathrm{VARS}(\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}))$. In conclusion, the induction hypothesis holds for this base case.

Unicast (T1-1): AWN defines the inference rule

$$\dfrac{}{\xi,\mathbf{unicast}(dest,ms).\mathrm{p} \blacktriangleright \mathrm{q} \xrightarrow{\mathbf{unicast}(\xi(dest),\xi(ms))} \xi,\mathrm{p}}\,\text{Unicast (T1-1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{unicast}(\xi(dest),\xi(ms)) \in A_1 \wedge \xi,\mathbf{unicast}(dest,ms).\mathrm{p} \xrightarrow{A_1} \xi,\mathrm{p}$, namely Pair 2.6 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi,\mathbf{unicast}(dest,ms).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi,\mathrm{p})$ for all $a \in A_2 = \{\ \mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(ms)]\!])\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{cast}(\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(ms)) \xrightarrow{[\![\mathbf{cast}(\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(ms))]\!]} \checkmark}\,\text{Axiom}}{\mathbf{cast}(\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(ms)) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(ms)]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{cast}(\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Seq 1}}{\mathbf{cast}(\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}) + \neg\mathbf{uni}(\mathrm{T}_\xi(\{dest\}),\emptyset,\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Choice 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(dest,ms).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{T3}}{\mathrm{T}(\xi,\mathbf{unicast}(dest,ms).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(\{dest\})]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}(\xi,\mathrm{p})}\,\text{T12}$$

In conclusion, the induction hypothesis holds for this base case.

Unicast (T1-2): AWN defines the inference rule

$$\dfrac{}{\xi,\mathbf{unicast}(dest,ms).\mathrm{p} \blacktriangleright \mathrm{q} \xrightarrow{\neg\mathbf{unicast}(\xi(dest),\xi(ms))} \xi,\mathrm{q}}\,\text{Unicast (T1-2)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\neg\mathbf{unicast}(\xi(dest),\xi(ms)) \in A_1 \wedge \xi,\mathbf{unicast}(dest,ms).\mathrm{p} \xrightarrow{A_1} \xi,\mathrm{p}$, namely Pair 2.7 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi,\mathbf{unicast}(dest,ms).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi,\mathrm{p})$ for all $a \in A_2 = \{\ \neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg\mathbf{uni}(\mathrm{T}_\xi(\{dest\}),\emptyset,\mathrm{T}_\xi(ms)) \xrightarrow{[\![\neg\mathbf{uni}(\mathrm{T}_\xi(\{dest\}),\emptyset,\mathrm{T}_\xi(ms))]\!]} \checkmark}\,\text{Axiom}}{\neg\mathbf{uni}(\mathrm{T}_\xi(\{dest\}),\emptyset,\mathrm{T}_\xi(ms)) \xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \checkmark}\,\text{Definition 4.10}}{\neg\mathbf{uni}(\mathrm{T}_\xi(\{dest\}),\emptyset,\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q}) \xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})}\,\text{Seq 1}}{\mathbf{cast}(\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(\{dest\}),\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}) + \neg\mathbf{uni}(\mathrm{T}_\xi(\{dest\}),\emptyset,\mathrm{T}_\xi(ms)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q}) \xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})}\,\text{Choice 4}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(dest,ms).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})}\,\text{T3}}{\mathrm{T}(\xi,\mathbf{unicast}(dest,ms).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}(\xi,\mathrm{q})}\,\text{T12}$$

In conclusion, the induction hypothesis holds for this base case.

Send (T1): AWN defines the inference rule

$$\frac{}{\xi, \mathbf{send}(ms).\mathrm{p} \xrightarrow{\mathbf{send}(\xi(ms))} \xi, \mathrm{p}}\ \textsc{Send (T1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{send}(\xi(ms)) \in A_1 \wedge \xi, \mathbf{send}(ms).\mathrm{p} \xrightarrow{A_1} \xi, \mathrm{p}$, namely Pair 2.8 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi, \mathbf{send}(ms).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi, \mathrm{p})$ for all $a \in A_2 = \{\ \mathbf{send}(\llbracket\emptyset\rrbracket, \llbracket\emptyset\rrbracket, \llbracket \mathrm{T}_\xi(ms)\rrbracket)\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{send}(\emptyset,\emptyset,\mathrm{T}_\xi(dest)) \xrightarrow{\llbracket \mathbf{send}(\emptyset,\emptyset,\mathrm{T}_\xi(ms))\rrbracket} \checkmark}\ \text{Axiom}}{\mathbf{send}(\emptyset,\emptyset,\mathrm{T}_\xi(dest)) \xrightarrow{\mathbf{send}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket \mathrm{T}_\xi(ms)\rrbracket)} \checkmark}\ \text{Definition 4.10}}{\mathbf{send}(\emptyset,\emptyset,\mathrm{T}_\xi(dest)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q}) \xrightarrow{\mathbf{send}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket \mathrm{T}_\xi(ms)\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\ \text{Seq 1}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{send}(ms).\mathrm{p}) \xrightarrow{\mathbf{send}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket \mathrm{T}_\xi(ms)\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\ \text{T4}}{\mathrm{T}(\xi, \mathbf{send}(ms).\mathrm{p}) \xrightarrow{\mathbf{send}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket \mathrm{T}_\xi(ms)\rrbracket)} \mathrm{T}(\xi,\mathrm{p})}\ \text{T12}$$

In conclusion, the induction hypothesis holds for this base case.

Deliver (T1): AWN defines the inference rule

$$\frac{}{\xi, \mathbf{deliver}(data).\mathrm{p} \xrightarrow{\mathbf{deliver}(\xi(data))} \xi, \mathrm{p}}\ \textsc{Deliver (T1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{deliver}(\xi(data)) \in A_1 \wedge \xi, \mathbf{deliver}(data).\mathrm{p} \xrightarrow{A_1} \xi, \mathrm{p}$, namely Pair 2.9 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi, \mathbf{deliver}(data).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi, \mathrm{p})$ for all $a \in A_2 = \{\ \mathbf{del}(\llbracket\mathtt{î}\rrbracket, \llbracket \mathrm{T}_\xi(data)\rrbracket) \mid \mathtt{î} : \mathrm{IP}\ \}$.
In mCRL2, the following derivation can be made for all $\mathtt{î}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{del}(\mathtt{î},\mathrm{T}_\xi(data)) \xrightarrow{\llbracket \mathbf{del}(\mathtt{î},\mathrm{T}_\xi(data))\rrbracket} \checkmark}\ \text{Axiom}}{\mathbf{del}(\mathtt{î},\mathrm{T}_\xi(data)) \xrightarrow{\mathbf{del}(\llbracket\mathtt{î}\rrbracket,\llbracket \mathrm{T}_\xi(data)\rrbracket)} \checkmark}\ \text{Definition 4.10}}{\mathbf{del}(\mathtt{î},\mathrm{T}_\xi(data)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q}) \xrightarrow{\mathbf{del}(\llbracket\mathtt{î}\rrbracket,\llbracket \mathrm{T}_\xi(data)\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\ \text{Seq 1}}{\mathbf{del}(\mathtt{ip},\mathrm{T}_\xi(data)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})\,[\mathtt{ip} := \mathtt{î}] \xrightarrow{\mathbf{del}(\llbracket\mathtt{î}\rrbracket,\llbracket \mathrm{T}_\xi(data)\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\ \text{Substitution}}{\sum_{\mathtt{ip}} \mathbf{del}(\mathtt{ip},\mathrm{T}_\xi(data)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q}) \xrightarrow{\mathbf{del}(\llbracket\mathtt{î}\rrbracket,\llbracket \mathrm{T}_\xi(data)\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\ \text{Sum 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{deliver}(ms).\mathrm{p}) \xrightarrow{\mathbf{del}(\llbracket\mathtt{î}\rrbracket,\llbracket \mathrm{T}_\xi(data)\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\ \text{T5}}{\mathrm{T}(\xi, \mathbf{deliver}(ms).\mathrm{p}) \xrightarrow{\mathbf{del}(\llbracket\mathtt{î}\rrbracket,\llbracket \mathrm{T}_\xi(data)\rrbracket)} \mathrm{T}(\xi,\mathrm{p})}\ \text{T12}$$

for $\mathtt{î} : \mathrm{Set(IP)}$ and $\mathtt{ip} \notin \textsc{vars}(\mathrm{T}_\xi(data)) \cup \textsc{vars}(\mathrm{T}_V(\xi,\mathrm{p}))$. In conclusion, the induction hypothesis holds for this base case.

Receive (T1): AWN defines the inference rule

$$\forall m \in \mathrm{MSG}\ \frac{}{\xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p} \xrightarrow{\mathbf{receive}(m)} \xi[\mathtt{msg} := m], \mathrm{p}}\ \textsc{Receive (T1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{receive}(m) \in A_1 \wedge \xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p} \xrightarrow{A_1} \xi[\mathtt{msg} := m], \mathrm{p}$, namely Pair 2.10 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi[\mathtt{msg} := m], \mathrm{p})$ for all $a \in A_2 = \{\ \mathbf{receive}(\llbracket\hat{\mathrm{D}}\rrbracket, \llbracket\hat{\mathrm{D}}\text{'}\rrbracket, \mathrm{U}(m)) \mid \hat{\mathrm{D}}, \hat{\mathrm{D}}\text{'} : \mathrm{Set(IP)}\ \}$.
In mCRL2, the following derivation can be made for all $\hat{\mathrm{D}}$, $\hat{\mathrm{D}}$':

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}) \xrightarrow{[\![\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)})]\!]} \checkmark}\,\text{Axiom}}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],[\![t_{\mathrm{U}(m)}]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],\mathrm{U}(m))} \checkmark}\,\text{Lemma 4.4}}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],\mathrm{U}(m))} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Seq 1}}{\big(\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},\mathrm{T}(\mathtt{msg})).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi,\mathrm{p})\big)\big[\mathrm{T}(\mathtt{msg}):=t_{\mathrm{U}(m)}\big] \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],\mathrm{U}(m))} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Lemma 4.3}}{\big(\mathbf{receive}(\mathtt{D},\mathtt{D}\text{'},\mathrm{T}(\mathtt{msg})).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi,\mathrm{p})\big)\big[\mathtt{D}:=\hat{\mathtt{D}},\mathtt{D}\text{'}:=\hat{\mathtt{D}}\text{'},\mathrm{T}(\mathtt{msg}):=t_{\mathrm{U}(m)}\big] \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],\mathrm{U}(m))} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Substitution}}{\sum_{\mathtt{D},\mathtt{D}\text{'}:\mathrm{Set(IP)},\mathrm{T}(\mathtt{msg}):\mathrm{sort}(\mathrm{T}(\mathtt{msg}))}\mathbf{receive}(\mathtt{D},\mathtt{D}\text{'},\mathrm{T}(\mathtt{msg})).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\mathtt{D}]\!],[\![\mathtt{D}\text{'}]\!],\mathrm{U}(m))} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Sum 2 (3 times)}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],\mathrm{U}(m))} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{T6}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],\mathrm{U}(m))} \mathrm{T}_{\mathrm{DOM}(\xi[\mathtt{msg}:=m])}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{T12}}{\mathrm{T}(\xi,\mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\text{'}]\!],\mathrm{U}(m))} \mathrm{T}(\xi[\mathtt{msg}:=m],\mathrm{p})}$$

for $\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'} : \mathrm{Set(IP)}$ and $\mathtt{D},\mathtt{D}\text{'} \notin \mathrm{VARS}(\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{msg}\}}(\xi,\mathrm{p}))$. In conclusion, the induction hypothesis holds for this base case.

Assignment (T1): AWN defines the inference rule

$$\dfrac{}{\xi,[\![\mathtt{var}:=\mathit{exp}]\!]\mathrm{p} \xrightarrow{\tau} \xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p}}\,\text{Assignment (T1)}$$

There exists an (A_1,A_2) in $\mathcal{A}$ such that $\tau \in A_1 \wedge \xi,[\![\mathtt{var}:=\mathit{exp}]\!]\mathrm{p} \xrightarrow{A_1} \xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p}$, namely Pair 2.3 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi,[\![\mathtt{var}:=\mathit{exp}]\!]\mathrm{p}) \xrightarrow{a} \mathrm{T}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})$ for all $a \in A_2 = \{\ \mathbf{t}(\mathrm{U}(D),\mathrm{U}(R),\mathrm{U}(m))\ \}$ for some $D,R : \mathrm{Set(IP)}$ where $R \subseteq D$ and some $m : \mathrm{MSG}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}) \xrightarrow{[\![\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}})]\!]} \checkmark}\,\text{Axiom}}{\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Seq 1}}{[\![t_{\mathrm{U}(\xi(\mathit{exp}))}=t_{\mathrm{U}(\xi(\mathit{exp}))}]\!]=\mathit{true}\quad (t_{\mathrm{U}(\xi(\mathit{exp}))}=t_{\mathrm{U}(\xi(\mathit{exp}))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Guard 2}}{(\mathrm{T}(\mathtt{var})=t_{\mathrm{U}(\xi(\mathit{exp}))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p})\big[\mathrm{T}(\mathtt{var}):=t_{\mathrm{U}(\xi(\mathit{exp}))}\big] \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Lemma 4.3}}{\Big((t_{\mathrm{U}(\xi(\mathit{exp}))}=t_{\mathrm{U}(\xi(\mathit{exp}))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p})\Big)\big[\mathrm{T}(\mathtt{var}):=t_{\mathrm{U}(\xi(\mathit{exp}))}\big] \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Sum 2}}{[\![t_{\mathrm{U}(\xi(\mathit{exp}))}=t_{\mathrm{U}(\xi(\mathit{exp}))}]\!]=\mathit{true}\quad \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var})=t_{\mathrm{U}(\xi(\mathit{exp}))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Guard 2}}{(t_{\mathrm{U}(\xi(\mathit{exp}))}=t_{\mathrm{U}(\xi(\mathit{exp}))}) \rightarrow \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var})=t_{\mathrm{U}(\xi(\mathit{exp}))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Lemma 4.1}}{(t_{\mathrm{U}(\xi(\mathit{exp}))}=\mathrm{T}_\xi(\mathit{exp})) \rightarrow \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var})=t_{\mathrm{U}(\xi(\mathit{exp}))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Sum 2}}{\Big((\mathtt{tmp}=\mathrm{T}_\xi(\mathit{exp})) \rightarrow \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var})=\mathtt{tmp}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p})\Big)\big[\mathtt{tmp}:=t_{\mathrm{U}(\xi(\mathit{exp}))}\big] \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{Sum 2}}{\sum_{\mathtt{tmp}:\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathtt{tmp}=\mathrm{T}_\xi(\mathit{exp})) \rightarrow \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var})=\mathtt{tmp}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\mathit{dummy}}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{T7}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,[\![\mathtt{var}:=\mathit{exp}]\!]\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}$$
$$\dfrac{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,[\![\mathtt{var}:=\mathit{exp}]\!]\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi[\mathtt{var}:=\xi(\mathit{exp})])}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}{\mathrm{T}(\xi,[\![\mathtt{var}:=\mathit{exp}]\!]\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\mathit{dummy}}]\!])} \mathrm{T}(\xi[\mathtt{var}:=\xi(\mathit{exp})],\mathrm{p})}\,\text{T12}$$

In conclusion, the induction hypothesis holds for this base case.

Guard (T1): AWN defines the inference rule

$$\zeta(\phi)=\mathit{true} \wedge \{\mathtt{q}_1,\cdots,\mathtt{q}_n\} = \mathrm{Fv}(\phi) \setminus \mathrm{DOM}(\xi)\ \dfrac{\zeta=\xi[\mathtt{q}_1:=e_1,\cdots,\mathtt{q}_n:=e_n]}{\xi,[\phi]\mathrm{p} \xrightarrow{\tau} \zeta,\mathrm{p}}\,\text{Guard (T1)}$$

There exists an (A_1,A_2) in $\mathcal{A}$ such that $\tau \in A_1 \wedge \xi,[\phi]\mathrm{p} \xrightarrow{A_1} \zeta,\mathrm{p}$, namely Pair 2.3 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}(\xi,[\phi]\mathrm{p}) \xrightarrow{a} \mathrm{T}(\zeta,\mathrm{p})$ for all $a \in A_2 = \left\{\ \mathbf{t}(\mathrm{U}(D),\mathrm{U}(R),[\![\mathtt{msg}_{\mathit{dummy}}]\!])\ \right\}$ for some $D,R : \mathrm{Set(IP)}$ where $R \subseteq D$ and some $m : \mathrm{MSG}$.
First, the side condition of the inference rule is used to determine that

$$\zeta(\phi)=\mathit{true} \xrightarrow{\text{Property (ii) of T}} [\![\mathrm{T}_\zeta(\phi)]\!]=\mathit{true}$$

With this side condition, the following derivation can be made in mCRL2:

$$\dfrac{}{\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}) \xrightarrow{\llbracket \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy})\rrbracket} \checkmark}\ \textsc{Axiom}$$

Definition 4.10

$$\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \checkmark$$

Seq 1

$$\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\zeta,\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\zeta,\mathrm{p})$$

$\llbracket \mathrm{T}_\zeta(\phi)\rrbracket = true$ — Guard 2

$$\mathrm{T}_\zeta(\phi) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\zeta,\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\zeta,\mathrm{p})$$

Definition of ζ (3 times)

$$\mathrm{T}_{\xi[\mathtt{q}_1:=e_1,\cdots,\mathtt{q}_n:=e_n]}(\phi) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\xi[\mathtt{q}_1:=e_1,\cdots,\mathtt{q}_n:=e_n],\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

Lemma 4.2

$$\mathrm{T}_\xi(\phi)[\mathrm{T}(\mathtt{q}_1):=t_{\mathrm{U}(e_1)},\cdots,\mathrm{T}(\mathtt{q}_n):=t_{\mathrm{U}(e_n)}] \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\xi[\mathtt{q}_1:=e_1,\cdots,\mathtt{q}_n:=e_n],\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

Lemma 4.3

$$\mathrm{T}_\xi(\phi)[\mathrm{T}(\mathtt{q}_1):=t_{\mathrm{U}(e_1)},\cdots,\mathrm{T}(\mathtt{q}_n):=t_{\mathrm{U}(e_n)}] \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\xi,\mathrm{p})[\mathrm{T}(\mathtt{q}_1):=t_{\mathrm{U}(e_1)},\cdots,\mathrm{T}(\mathtt{q}_n):=t_{\mathrm{U}(e_n)}] \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

$$\Big(\mathrm{T}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\xi,\mathrm{p})\Big)[\mathrm{T}(\mathtt{q}_1):=t_{\mathrm{U}(e_1)},\cdots,\mathrm{T}(\mathtt{q}_n):=t_{\mathrm{U}(e_n)}] \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

Sum 2 (n times)

$$\textstyle\sum_{\{\mathrm{T}(\mathtt{q}_1),\cdots,\mathrm{T}(\mathtt{q}_n)\}} \mathrm{T}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

$$\textstyle\sum_{\mathrm{T}(\{\mathtt{q}_1,\cdots,\mathtt{q}_n\})} \mathrm{T}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

Definition of $\{\mathtt{q}_1,\cdots,\mathtt{q}_n\}$ (2 times)

$$\textstyle\sum_{\mathrm{T}(\mathrm{Fv}(\phi)\,\setminus\,\mathrm{DOM}(\xi))} \mathrm{T}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup(\mathrm{Fv}(\phi)\,\setminus\,\mathrm{DOM}(\xi))}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

$$\textstyle\sum_{\mathrm{T}(\mathrm{Fv}(\phi)\,\setminus\,\mathrm{DOM}(\xi))} \mathrm{T}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\mathrm{Fv}(\phi)}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

T10

$$\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,[\phi]\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta,\mathrm{p})$$

T12

$$\mathrm{T}(\xi,[\phi]\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\,dummy}\rrbracket)} \mathrm{T}(\zeta,\mathrm{p})$$

In conclusion, the induction hypothesis holds for this base case.

Arrive (T3-2): AWN defines the inference rule

$$\dfrac{}{ip:\mathrm{P}:R \xrightarrow{\emptyset\neg\{ip\}:\mathbf{arrive}(m)} ip:\mathrm{P}:R}\ \textsc{Arrive (T3-2)}$$

There exists an (A_1,A_2) in $\mathcal{A}$ such that $\emptyset\neg\{ip\}:\mathbf{arrive}(m) \in A_1 \wedge ip:\mathrm{P}:R \xrightarrow{A_1} ip:\mathrm{P}':R$, namely Pair 2.13 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 for $\mathrm{T}(ip:\mathrm{P}:R) \xrightarrow{a} \mathrm{T}(ip:\mathrm{P}':R)$ for all $a \in A_2$ where

$$A_2 = \left\{ \mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m)) \;\middle|\; \begin{matrix}\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'}:\mathrm{Set(IP)}\\ \hat{\mathtt{D}}\text{'}\subseteq\hat{\mathtt{D}}\\ \{ip\}\cap\hat{\mathtt{D}}\text{'}=\emptyset\end{matrix} \right\}$$

Note that $\{ip\}\cap\hat{\mathtt{D}}\text{'} = \emptyset \Rightarrow \llbracket t_{\mathrm{U}(ip)} \notin \hat{\mathtt{D}}\text{'}\rrbracket = true$
In mCRL2, the following derivation can be made for all $\hat{\mathtt{D}}$ and $\hat{\mathtt{D}}\text{'}$ such that $\hat{\mathtt{D}}\text{'} \subseteq \hat{\mathtt{D}} \wedge t_{\mathrm{U}(ip)} \notin \hat{\mathtt{D}}\text{'}$:

$$\dfrac{}{\mathbf{arrive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}) \xrightarrow{\llbracket\mathbf{arrive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)})\rrbracket} \checkmark}\ \textsc{Axiom}$$

Definition 4.10

$$\mathbf{arrive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\llbracket t_{\mathrm{U}(m)}\rrbracket)} \checkmark$$

Lemma 4.4

$$\mathbf{arrive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \checkmark$$

Seq 1

$$\mathbf{arrive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

$\llbracket t_{\mathrm{U}(ip)} \notin \hat{\mathtt{D}}\text{'}\rrbracket = true$ — Guard 2

$$(t_{\mathrm{U}(ip)} \notin \hat{\mathtt{D}}\text{'}) \rightarrow \mathbf{arrive}(\hat{\mathtt{D}},\hat{\mathtt{D}}\text{'},t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

Substitution

$$\big(\,(t_{\mathrm{U}(ip)} \notin \mathtt{D'}) \rightarrow \mathbf{arrive}(\mathtt{D},\mathtt{D'},\mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})\,\big)\,[\,\mathtt{D}:=\hat{\mathtt{D}},\mathtt{D'}:=\hat{\mathtt{D}}\text{'},\mathtt{msg}:=t_{\mathrm{U}(m)}\,] \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

Sum 2 (3 times)

$$\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(t_{\mathrm{U}(ip)} \notin \mathtt{D'}) \rightarrow \mathbf{arrive}(\mathtt{D},\mathtt{D'},\mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

Choice 2

$$\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(t_{\mathrm{U}(ip)} \notin \mathtt{D'}) \rightarrow \mathbf{arrive}(\mathtt{D},\mathtt{D'},\mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}) + \mathrm{S} \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

Substitution

$$\Big(\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(\mathtt{ip} \notin \mathtt{D'}) \rightarrow \mathbf{arrive}(\mathtt{D},\mathtt{D'},\mathtt{msg}).\mathrm{G}(\mathtt{ip},\mathtt{R}) + \mathrm{S}\Big)\,[\,\mathtt{ip}:=t_{\mathrm{U}(ip)},\mathtt{R}:=t_{\mathrm{U}(R)}\,] \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

Definition of G — Recursion 2

$$\mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

Par 5

$$\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})$$

Comm 2

$$\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})) \xrightarrow{\gamma_C(\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m)))} \Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}))$$

Apply γ_C

$$\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}))$$

Rename 2

$$\rho_{\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}}\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}\bullet\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \rho_{\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}}\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}))$$

Apply $\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}\bullet$

$$\rho_{\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}}\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \rho_{\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}}\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}))$$

$\mathbf{arrive} \in V \cup \{\tau\}$ — Allow 2

$$\nabla_V\rho_{\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}}\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \nabla_V\rho_{\{\neg\mathbf{unicast}\rightarrow\mathbf{t}\}}\Gamma_C(\mathrm{T(P)} \,||\, \mathrm{G}(t_{\mathrm{U}(ip)},t_{\mathrm{U}(R)}))$$

T14

$$\mathrm{T}(ip:\mathrm{P}:R) \xrightarrow{\mathbf{arrive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\text{'}\rrbracket,\mathrm{U}(m))} \mathrm{T}(ip:\mathrm{P}:R)$$

where $\mathrm{S}[\;\mathtt{ip}:=t_{\mathrm{U}(ip)},\mathtt{R}:=t_{\mathrm{U}(R)}\;]$ is an expression equal to all summands of G except for the one containing **arrive**.

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P} : R) \text{ for all } a \in A_2 = \left\{ \mathbf{arrive}([\![\hat{\mathtt{D}}]\!], [\![\hat{\mathtt{D}}\text{'}]\!], \mathrm{U}(m)) \;\middle|\; \begin{smallmatrix} \hat{\mathtt{D}},\hat{\mathtt{D}}\text{'}:\mathrm{Set(IP)} \\ \hat{\mathtt{D}}\text{'}\subseteq\hat{\mathtt{D}} \\ \{ip\}\cap\hat{\mathtt{D}}\text{'}=\emptyset \end{smallmatrix} \right\}$$

which finishes this particular base case.

Connect (T3-1): AWN defines the inference rule

$$\frac{}{ip : \mathrm{P} : R \xrightarrow{\mathbf{connect}(ip, ip')} ip : \mathrm{P} : R \cup \{ip'\}}\ \textsc{Connect (T3-1)}$$

There exist an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{connect}(ip, ip') \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R \cup \{ip'\}$, namely Pair 2.14 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R \cup \{ip'\})$ for all $a \in A_2 = \{\ \mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{connect}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(ip')}) \xrightarrow{[\![\mathbf{connect}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(ip')})]\!]} \checkmark}\ \textsc{Axiom}}{\mathbf{connect}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(ip')}) \xrightarrow{\mathbf{connect}([\![t_{\mathrm{U}(ip)}]\!], [\![t_{\mathrm{U}(ip')}]\!])} \checkmark}\ \text{Definition 4.10}}{\mathbf{connect}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(ip')}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \checkmark}\ \text{Lemma 4.4}}{\mathbf{connect}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(ip')}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})}\ \textsc{Seq 1}}{(\ \mathbf{connect}(t_{\mathrm{U}(ip)}, \mathtt{ip'}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})\ )[\ \mathtt{ip'} := t_{\mathrm{U}(ip')}\ ] \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})}\ \text{Substitution}}{\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(t_{\mathrm{U}(ip)}, \mathtt{ip'}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})}\ \textsc{Sum 2}}{\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(t_{\mathrm{U}(ip)}, \mathtt{ip'}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}) + \mathrm{S}[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\ ] \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})}\ \textsc{Choice 2}}{\left(\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(\mathtt{ip}, \mathtt{ip'}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S}\right)[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)} \cup \{ip'\}\ ] \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})}\ \text{Substitution}}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})}\ \textsc{Recursion 2}\ \text{Definition of G}}{\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\})}\ \textsc{Par 5}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\gamma_C(\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip')))} \Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}))}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}))}\ \textsc{Rename 2}}{\mathbf{connect} \in V \cup \{\tau\}}\ \text{Apply } \{\neg\mathbf{unicast} \to \mathbf{t}\}\bullet$$

$$\dfrac{\dfrac{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}))}{\nabla_V \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \nabla_V \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)} \cup \{ip'\}))}\ \textsc{T14}}{\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))} \mathrm{T}(ip : \mathrm{P} : R \cup \{ip'\})}\ \textsc{Allow 2}$$

where $\mathrm{S}[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\ ]$ is an expression equal to all summands of G except for the first summand containing **connect**.
So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P} : R \cup \{ip'\}) \text{ for all } a \in A_2 = \{\ \mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))\ \}$$

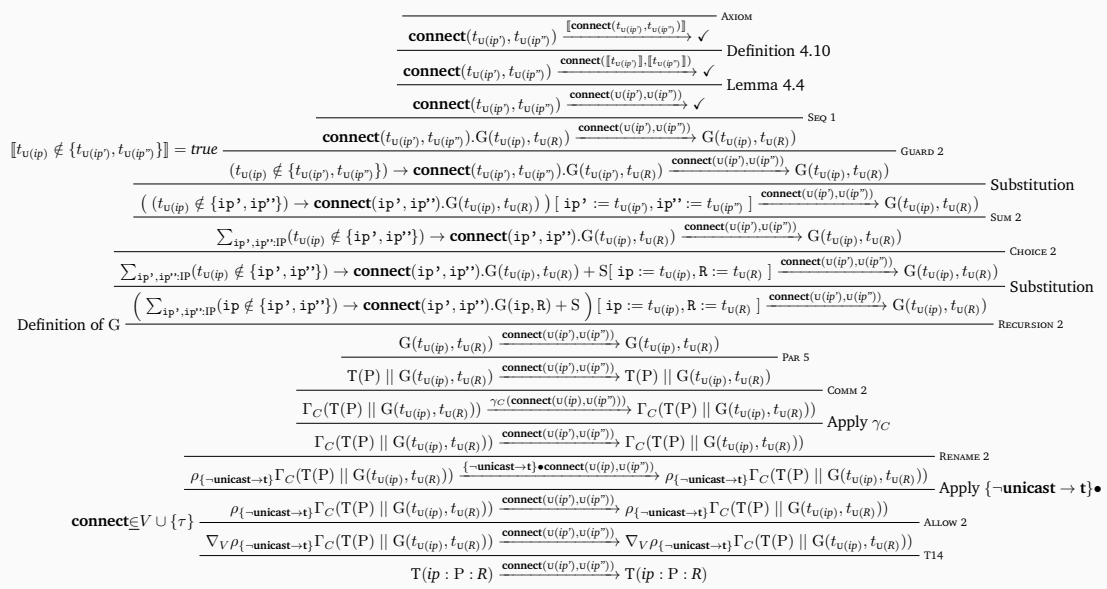
which finishes this particular base case.

Connect (T3-2): Similar to the Lemma 4.6-proof for Connect (T3-1).

Connect (T3-3): AWN defines the inference rule

$$\frac{ip \notin \{ip', ip''\}}{ip : \mathrm{P} : R \xrightarrow{\mathbf{connect}(ip', ip'')} ip : \mathrm{P} : R}\ \textsc{Connect (T3-3)}$$

There exist an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{connect}(ip', ip'') \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R$, namely Pair 2.14 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R)$ for all $a \in A_2 = \{\ \mathbf{connect}(\mathrm{U}(ip'), \mathrm{U}(ip''))\ \}$.
In mCRL2, the following derivation can be made:



where $\mathrm{S}[\ \mathtt{ip} := t_{\mathtt{U}(ip)}, \mathtt{R} := t_{\mathtt{U}(R)}\ ]$ is an expression equal to all summands of G except for the third summand containing **connect**.
So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P} : R) \text{ for all } a \in A_2 = \{\ \mathbf{connect}(\mathtt{U}(ip'), \mathtt{U}(ip''))\ \} \text{ with } ip \notin \{ip', ip''\}$$

which finishes this particular base case.

Disconnect (T3-1): Similar to the Lemma 4.6-proof for Connect (T3-1).

Disconnect (T3-2): Similar to the Lemma 4.6-proof for Connect (T3-1).

Disconnect (T3-3): Similar to the Lemma 4.6-proof for Connect (T3-3).

2 Induction steps

Recursion (T1): AWN defines the inference rule

$$\forall a \in \mathrm{Act}\ \frac{\emptyset[\mathtt{var}_i := \xi(exp_i)]_{i=1}^{n}, \mathrm{p} \xrightarrow{a} \zeta, \mathrm{p}' \qquad \mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}}{\xi, \mathrm{X}(exp_1, \cdots, exp_n) \xrightarrow{a} \zeta, \mathrm{p}'}\ \textsc{Recursion (T1)}$$

According to the induction hypothesis,

$$\exists (A_1, A_2) \in \mathcal{A}\ .\ a \in A_1 \wedge \emptyset[\mathtt{var}_i := \xi(exp_i)]_{i=1}^{n}, \mathrm{p} \xrightarrow{A_1} \zeta, \mathrm{p}' \wedge \mathrm{T}(\emptyset[\mathtt{var}_i := \xi(exp_i)]_{i=1}^{n}, \mathrm{p}) \xrightarrow{A_2} \mathrm{T}(\zeta, \mathrm{p}')$$

and so the following derivation can be made for all $a' \in A_2$:

$$\mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \overset{\text{def}}{=} \mathrm{p},\ \text{T11}\ \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\text{Induction hypothesis}}}{\mathrm{T}(\emptyset[\mathtt{var}_i := \xi(exp_i)]_{i=1}^n, \mathrm{p}) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \text{T12}}{\mathrm{T}_{\{\mathtt{var}_1, \cdots, \mathtt{var}_n\}}(\emptyset[\mathtt{var}_i := \xi(exp_i)]_{i=1}^n, \mathrm{p}) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \text{Lemma 4.3}}{\mathrm{T}_{\{\mathtt{var}_1, \cdots, \mathtt{var}_n\}}(\emptyset, \mathrm{p}) \left[\mathrm{T}(\mathtt{var}_i) := t_{\mathrm{U}(\xi(exp_i))}\right]_{i=1}^n \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \textsc{Recursion 2}}{\mathrm{X}(t_{\mathrm{U}(\xi(exp_1))}, \cdots, t_{\mathrm{U}(\xi(exp_n))}) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \text{Lemma 4.1}}{\mathrm{X}(\mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}_\xi(exp_n)) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \text{T8}}{\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \text{Choose } V = \textsc{dom}(\xi)}{\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \text{T12}}{\mathrm{T}(\mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}$$

This proves this particular induction step.

Choice (T1-1): AWN defines the inference rule

$$\forall a \in \mathrm{Act}\ \frac{\xi, \mathrm{p} \xrightarrow{a} \zeta, \mathrm{p}'}{\xi, \mathrm{p} + \mathrm{q} \xrightarrow{a} \zeta, \mathrm{p}'}\ \textsc{Choice (T1-1)}$$

According to the induction hypothesis,

$$\exists (A_1, A_2) \in \mathcal{A}\ .\ a \in A_1 \wedge \xi, \mathrm{p} \xrightarrow{A_1} \zeta, \mathrm{p}' \wedge \mathrm{T}(\xi, \mathrm{p}) \xrightarrow{A_2} \mathrm{T}(\zeta, \mathrm{p}')$$

and so the following derivation can be made for all $a' \in A_2$:

$$\dfrac{\dfrac{\dfrac{}{\text{Induction hypothesis}}}{\mathrm{T}(\xi, \mathrm{p}) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \textsc{Choice 2}}{\mathrm{T}(\xi, \mathrm{p}) + \mathrm{T}(\xi, \mathrm{q}) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}\ \text{T9}}{\mathrm{T}(\xi, \mathrm{p} + \mathrm{q}) \xrightarrow{a'} \mathrm{T}(\zeta, \mathrm{p}')}$$

This proves this particular induction step.

Choice (T1-2): Similar to the the Lemma 4.6-proof for Choice (T1-1) (Choice 4 is used instead of Choice 2).

Parallel (T2-1): AWN defines the inference rule

$$\forall a \neq \textbf{receive}(m)\ \frac{\mathrm{P} \xrightarrow{a} \mathrm{P}'}{\mathrm{P} \between \mathrm{Q} \xrightarrow{a} \mathrm{P}' \between \mathrm{Q}}\ \textsc{Parallel (T2-1)}$$

According to the induction hypothesis,

$$\exists (A_1, A_2) \in \mathcal{A}\ .\ a \in A_1 \wedge \mathrm{P} \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_2} \mathrm{T}(\mathrm{P}')$$

Because $a \neq$ **receive**, Pair 2.10 from Table 2.6 cannot be among the pairs (A_1, A_2) that satisfy the induction hypothesis (of which there must be at least one). Furthermore, according to the AWN semantics a must be an action that can be performed at the sequential level, meaning that $a \in \{\tau, \textbf{broadcast}, \textbf{groupcast}, \textbf{unicast}, \textbf{send}, \textbf{deliver}\}$. As a consequence,

$$\forall (A_1, A_2) \in \mathcal{A}\ .\ a \in A_1 \wedge \mathrm{P} \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_2} \mathrm{T}(\mathrm{P}') \Rightarrow \forall a' \in A_2\ .\ \underline{a'} \in W$$
$$\text{where } W = \{\textbf{t}, \textbf{cast}, \textbf{send}, \textbf{del}\}$$

Given this, the following derivation can be made in mCRL2 for all $a' \in A_2$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{a'} \mathrm{T}(\mathrm{P}')}\text{ Induction hypothesis}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{a'} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\text{ Rename 2}}{\underline{a'} \neq \mathbf{receive}\ \dfrac{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\{\mathbf{receive}\to\mathbf{r}\}\bullet a'} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{a'} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\text{ Apply }\{\mathbf{receive}\to\mathbf{r}\}\bullet}\text{ Par 2}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{a'} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})}\text{ Comm 2}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(a')} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ Apply }\gamma}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a'} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ Rename 2}}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\{\mathbf{rs}\to\mathbf{t}\}\bullet a'} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ Apply }\{\mathbf{rs}\to\mathbf{t}\}\bullet}{\underline{a'}\in V\cup\{\tau\}\supseteq W\ \dfrac{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a'} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}{\nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a'} \nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ T13}}\text{ Allow 2}}{\mathrm{T}(\mathrm{P}\ll\mathrm{Q}) \xrightarrow{a'} \mathrm{T}(\mathrm{P}'\ll\mathrm{Q})}$$

This proves this particular induction step.

Parallel (T2-2): AWN defines the inference rule

$$\forall a \neq \mathbf{send}(m)\ \frac{\mathrm{Q} \xrightarrow{a} \mathrm{Q}'}{\mathrm{P}\ll\mathrm{Q} \xrightarrow{a} \mathrm{P}\ll\mathrm{Q}'}\ \text{Parallel (T2-2)}$$

According to the induction hypothesis,

$$\exists(A_1, A_2) \in \mathcal{A}\ .\ a \in A_1 \wedge \mathrm{Q} \xrightarrow{A_1} \mathrm{Q}' \wedge \mathrm{T}(\mathrm{Q}) \xrightarrow{A_2} \mathrm{T}(\mathrm{Q}')$$

Because $a \neq$ **send**, Pair 2.8 from Table 2.6 cannot be among the pairs (A_1, A_2) that satisfy the induction hypothesis (of which there must be at least one). Furthermore, according to the AWN semantics a must be an action that can be performed at the sequential level, meaning that $a \in \{\tau, \mathbf{broadcast}, \mathbf{groupcast}, \mathbf{unicast}, \mathbf{deliver}, \mathbf{receive}\}$. As a consequence,

$$\forall(A_1, A_2) \in \mathcal{A}\ .\ a \in A_1 \wedge \mathrm{P} \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_2} \mathrm{T}(\mathrm{P}') \Rightarrow \forall a' \in A_2\ .\ \underline{a'} \in W$$
$$\text{where } W = \{\mathbf{t}, \mathbf{cast}, \mathbf{del}, \mathbf{receive}\}$$

Given this, the following derivation can be made in mCRL2 for all $a' \in A_2$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{Q}) \xrightarrow{a'} \mathrm{T}(\mathrm{Q}')}\text{ Induction hypothesis}}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{a'} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\text{ Rename 2}}{\underline{a'} \neq \mathbf{send}\ \dfrac{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\{\mathbf{send}\to\mathbf{s}\}\bullet a'} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{a'} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\text{ Apply }\{\mathbf{send}\to\mathbf{s}\}\bullet}\text{ Par 5}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{a'} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\text{ Comm 2}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(a')} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\text{ Apply }\gamma}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a'} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\text{ Rename 2}}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\{\mathbf{rs}\to\mathbf{t}\}\bullet a'} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\text{ Apply }\{\mathbf{rs}\to\mathbf{t}\}\bullet}{\underline{a'}\in V\cup\{\tau\}\supseteq W\ \dfrac{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a'} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}{\nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a'} \nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\text{ T13}}\text{ Allow 2}}{\mathrm{T}(\mathrm{P}\ll\mathrm{Q}) \xrightarrow{a'} \mathrm{T}(\mathrm{P}\ll\mathrm{Q}')}$$

This proves this particular induction step.

Parallel (T2-3): AWN defines the inference rule

$$\forall m \in \mathrm{MSG}\ \frac{\mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}' \qquad \mathrm{Q} \xrightarrow{\mathbf{send}(m)} \mathrm{Q}'}{\mathrm{P}\ll\mathrm{Q} \xrightarrow{\tau} \mathrm{P}'\ll\mathrm{Q}'}\ \text{Parallel (T2-3)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\tau \in A_1 \wedge \mathrm{P} \ll \mathrm{Q} \xrightarrow{A_1} \mathrm{P}' \ll \mathrm{Q}'$, namely Pair 2.3 in Table 2.6. The

induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(\mathrm{P} \lll \mathrm{Q}) \xrightarrow{a} \mathrm{T}(\mathrm{P}' \lll \mathrm{Q})$ for all $a \in A_2 = \left\{ \mathbf{t}(\mathtt{u}(D), \mathtt{u}(R), [\![\mathtt{msg}_{\,dummy}]\!]) \right\}$ for some D, R where $R \subseteq D$.

From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{receive}([\![\mathtt{D}]\!],[\![\mathtt{D'}]\!],\mathtt{u}(m))} \mathrm{T}(\mathrm{P}')$ for all `D`, `D'` because Pair 2.10 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{receive}(m) \in B_1$. Similarly, $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{send}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \mathrm{T}(\mathrm{P}')$ because Pair 2.8 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{send}(m) \in B_1$ and the following condition holds:

$$[\![\mathrm{T}_\xi(ms)]\!] \overset{\text{Lemma 4.1}}{=} [\![t_{\mathtt{u}(\xi(ms))}]\!] \overset{\text{Premises}}{=} [\![t_{\mathtt{u}(m)}]\!] \overset{\text{Lemma 4.4}}{=} \mathtt{u}(m)$$

This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\mathtt{'}]\!],\mathtt{u}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{receive}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Choose } \hat{\mathtt{D}} = \hat{\mathtt{D}}\mathtt{'} = \emptyset}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\{\mathbf{receive}\to\mathbf{r}\}\bullet\mathbf{receive}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\ \textsc{Rename 2}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\ \text{Apply } \{\mathbf{receive}\to\mathbf{r}\}\bullet$$

$$\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{send}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \mathrm{T}(\mathrm{Q}')}\ \text{Induction hypothesis}}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\{\mathbf{send}\to\mathbf{s}\}\bullet\mathbf{send}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \textsc{Rename 2}}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \text{Apply } \{\mathbf{send}\to\mathbf{s}\}\bullet$$

$$\mathbf{t} \trianglelefteq V \cup \{\tau\} \supseteq W \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\ \textit{(see above)} \quad \dfrac{}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \textit{(see above)}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))|\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \textsc{Par 3}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))|\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m)))} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \textsc{Comm 2}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\mathbf{rs}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \text{Apply } \gamma}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\mathbf{rs}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \textsc{Rename 2}}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\{\mathbf{rs}\to\mathbf{t}\}\bullet\mathbf{rs}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \text{Apply } \{\mathbf{rs}\to\mathbf{t}\}\bullet}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \textsc{Allow 2}}{\nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \textsc{T13}}{\mathrm{T}(\mathrm{P} \lll \mathrm{Q}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],\mathtt{u}(m))} \mathrm{T}(\mathrm{P}' \lll \mathrm{Q}')}$$

This proves this particular induction step.

Broadcast (T3): AWN defines the inference rule

$$\dfrac{\mathrm{P} \xrightarrow{\mathbf{broadcast}(m)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{R:\mathbf{*cast}(m)} ip : \mathrm{P}' : R}\ \textsc{Broadcast (T3)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $R : \mathbf{*cast}(m) \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R$, namely Pair 2.11 in Table 2.6 (note that it is possible to choose $D = \mathcal{U}_{\mathrm{IP}}$). The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R)$ for all $a \in A_2 = \{\ \mathbf{starcast}([\![\mathcal{U}_{\mathrm{IP}}]\!], \mathtt{u}(R), \mathtt{u}(m))\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \xrightarrow{\llbracket \overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \rrbracket} \checkmark}\ \text{Axiom}}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket t_{\mathrm{U}(R)} \rrbracket, \llbracket t_{\mathrm{U}(m)} \rrbracket)} \checkmark}\ \text{Definition 4.10}}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \checkmark}\ \text{Lemma 4.4}}{\overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Seq 1}}{\llbracket t_{\mathrm{U}(R)} \cap \mathcal{U}_{\mathrm{IP}} = t_{\mathrm{U}(R)} \rrbracket = \mathit{true} \quad (t_{\mathrm{U}(R)} \cap \mathcal{U}_{\mathrm{IP}} = t_{\mathrm{U}(R)}) \to \overline{\mathbf{cast}}(\mathcal{U}_{\mathrm{IP}}, t_{\mathrm{U}(R)}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Guard 2}}{(t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\big((t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \big) [\, \mathtt{D} := \mathcal{U}_{\mathrm{IP}}, \mathtt{D'} := t_{\mathrm{U}(R)}, \mathtt{msg} := t_{\mathrm{U}(m)} \,] \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Sum 2 (3 times)}}{\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Choice 2}}{\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) + \mathrm{S}[\, \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)} \,] \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\text{Definition of G} \quad \Big(\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (\mathtt{R} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S} \Big) [\, \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)} \,] \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Recursion 2}}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}$$

where $\mathrm{S}[\, \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)} \,]$ is an expression equal to all summands of G except for the first one.

From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))}$ because Pair 2.4 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{broadcast}(m) \in B_1$. Combining this with the conclusion of the derivation above gives

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{P'})}\ \text{Induction hypothesis} \quad \dfrac{}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textit{(see above)}}{\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m)) | \overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{P'}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\gamma_C(\mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m)) | \overline{\mathbf{cast}}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{P'}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \Gamma_C(\mathrm{T}(\mathrm{P'}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\{\neg \mathbf{unicast} \to \mathbf{t}\} \bullet \mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{P'}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Rename 2}}{\mathbf{starcast} \subseteq V \cup \{\tau\} \quad \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{P'}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Allow 2}}{\nabla_V \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \nabla_V \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{P'}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{T14}}{\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{\mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(ip : \mathrm{P'} : R)}\ \text{Apply } \{\neg \mathbf{unicast} \to \mathbf{t}\} \bullet$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P'} : R) \text{ for all } a \in A_2 = \{\ \mathbf{starcast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \mathrm{U}(R), \mathrm{U}(m))\ \}$$

which finishes the induction step for the case of the Broadcast (T3) inference rule.

Groupcast (T3): AWN defines the inference rule

$$\dfrac{\mathrm{P} \xrightarrow{\mathbf{groupcast}(D,m)} \mathrm{P'}}{ip : \mathrm{P} : R \xrightarrow{R \cap D : \mathbf{*cast}(m)} ip : \mathrm{P'} : R}\ \text{Groupcast (T3)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $R \cap D : \mathbf{*cast}(m) \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P'} : R$, namely Pair 2.11 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P'} : R)$ for all $a \in A_2 = \{\ \mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\mathbf{cast}}(t_{\mathrm{U}(D)}, t_{\mathrm{U}(R \cap D)}, t_{\mathrm{U}(m)}) \xrightarrow{\llbracket \overline{\mathbf{cast}}(t_{\mathrm{U}(D)}, t_{\mathrm{U}(R \cap D)}, t_{\mathrm{U}(m)}) \rrbracket} \checkmark}\ \text{Axiom}}{\overline{\mathbf{cast}}(t_{\mathrm{U}(D)}, t_{\mathrm{U}(R \cap D)}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(\llbracket t_{\mathrm{U}(D)} \rrbracket, \llbracket t_{\mathrm{U}(R \cap D)} \rrbracket, \llbracket t_{\mathrm{U}(m)} \rrbracket)} \checkmark}\ \text{Definition 4.10}}{\overline{\mathbf{cast}}(t_{\mathrm{U}(D)}, t_{\mathrm{U}(R \cap D)}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \checkmark}\ \text{Lemma 4.4}}{\overline{\mathbf{cast}}(t_{\mathrm{U}(D)}, t_{\mathrm{U}(R \cap D)}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Seq 1}}{\llbracket t_{\mathrm{U}(R)} \cap t_{\mathrm{U}(D)} = t_{\mathrm{U}(R \cap D)} \rrbracket = \mathit{true} \quad (t_{\mathrm{U}(R)} \cap t_{\mathrm{U}(D)} = t_{\mathrm{U}(R \cap D)}) \to \overline{\mathbf{cast}}(t_{\mathrm{U}(D)}, t_{\mathrm{U}(R \cap D)}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Guard 2}}{(t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\big((t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \big) [\, \mathtt{D} := t_{\mathrm{U}(D)}, \mathtt{D'} := t_{\mathrm{U}(R \cap D)}, \mathtt{msg} := t_{\mathrm{U}(m)} \,] \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Sum 2 (3 times)}}{\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Choice 2}}{\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) + \mathrm{S}[\, \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)} \,] \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\text{Definition of G} \quad \Big(\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (\mathtt{R} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S} \Big) [\, \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)} \,] \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Recursion 2}}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}$$

where $\mathrm{S}[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\ ]$ is an expression equal to all summands of G except for the first one.

From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \mathrm{T}(\mathrm{P}')$ because Pair 2.5 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{groupcast}(D, m) \in B_1$. Combining this with the conclusion of the derivation above gives

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis} \quad \dfrac{}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textit{(see above)}}{\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{cast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))|\overline{\mathbf{cast}}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\gamma_C(\mathbf{cast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))|\overline{\mathbf{cast}}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet\mathbf{starcast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{Rename 2}}{\mathbf{starcast} \in V \cup \{\tau\}\quad \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Apply } \{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet}{\nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{Allow 2}}{\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D),\mathrm{U}(R\cap D),\mathrm{U}(m))} \mathrm{T}(ip : \mathrm{P}' : R)}\ \textsc{T14}$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R) \text{ for all } a \in A_2 = \{\ \mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R \cap D), \mathrm{U}(m))\ \}$$

which finishes the induction step for the case of the GROUPCAST (T3) inference rule.

Unicast (T3-1): AWN defines the inference rule

$$\dfrac{\mathrm{P} \xrightarrow{\mathbf{unicast}(dip,m)} \mathrm{P}' \qquad dip \in R}{ip : \mathrm{P} : R \xrightarrow{\{dip\}:\mathbf{*cast}(m)} ip : \mathrm{P}' : R}\ \textsc{Unicast (T3-1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\{dip\} : \{ip\} : \mathbf{*cast}(m) \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R$, namely Pair 2.11 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R)$ for all $a \in A_2 = \{\ \mathbf{starcast}(\mathrm{U}(\{dip\}), \mathrm{U}(\{dip\}), \mathrm{U}(m))\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\mathbf{cast}}(t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(m)})} \checkmark}\ \textsc{Axiom}}{\overline{\mathbf{cast}}(t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}([\![t_{\mathrm{U}(\{dip\})}]\!], [\![t_{\mathrm{U}(\{dip\})}]\!], [\![t_{\mathrm{U}(m)}]\!])} \checkmark}\ \text{Definition 4.10}}{\overline{\mathbf{cast}}(t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}([\![t_{\mathrm{U}(\{dip\})}]\!], [\![t_{\mathrm{U}(R\cap D)}]\!], [\![t_{\mathrm{U}(m)}]\!])} \checkmark}\ \text{Lemma 4.4}}{\overline{\mathbf{cast}}(t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(m)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \checkmark}\ \textsc{Seq 1}}{[\![t_{\mathrm{U}(R)} \cap t_{\mathrm{U}(\{dip\})} = t_{\mathrm{U}(\{dip\})}]\!] = \mathit{true} \quad \overline{\mathbf{cast}}(t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Guard 2}}{(t_{\mathrm{U}(R)} \cap t_{\mathrm{U}(\{dip\})} = t_{\mathrm{U}(\{dip\})}) \to \overline{\mathbf{cast}}(t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(\{dip\})}, t_{\mathrm{U}(m)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\big(\ (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})\ \big)\ [\ \mathtt{D} := t_{\mathrm{U}(\{dip\})}, \mathtt{D'} := t_{\mathrm{U}(\{dip\})}, \mathtt{msg} := t_{\mathrm{U}(m)}\ ] \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Sum 2}\ \text{(2 times)}}{\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Choice 2}}{\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (t_{\mathrm{U}(R)} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) + \mathrm{S}[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\ ] \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\text{Definition of G}\ \dfrac{\Big(\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set}(\mathrm{IP}),\mathtt{msg}:\mathrm{MSG}} (\mathtt{R} \cap \mathtt{D} = \mathtt{D'}) \to \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S}\Big)\ [\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\ ] \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Recursion 2}}$$

where $\mathrm{S}[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\ ]$ is an expression equal to all summands of G except for the first one.

From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\mathrm{U}(\{dip\}),\mathrm{U}(\{dip\}),\mathrm{U}(m))} \mathrm{T}(\mathrm{P}')$ because Pair 2.6 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{unicast}(dip, m) \in B_1$. Combining this with the conclusion of the derivation above gives

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{cast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis} \quad \dfrac{}{\mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) \xrightarrow{\overline{\mathbf{cast}}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \textit{(see above)}}{\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) \xrightarrow{\mathbf{cast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))\mid\overline{\mathbf{cast}}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))} \mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\gamma_C(\mathbf{cast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))\mid\overline{\mathbf{cast}}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\mathbf{starcast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet\mathbf{starcast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{Rename 2}}{\mathbf{starcast}\in V\cup\{\tau\}\quad \nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\mathbf{starcast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))} \nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{T14}$$

$$\mathrm{T}(ip:\mathrm{P}:R) \xrightarrow{\mathbf{starcast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))} \mathrm{T}(ip:\mathrm{P}':R)$$

So it is indeed the case that

$$\mathrm{T}(ip:\mathrm{P}:R) \xrightarrow{a} \mathrm{T}(ip:\mathrm{P}':R) \text{ for all } a \in A_2 = \{\ \mathbf{starcast}(\mathtt{U}(\{dip\}),\mathtt{U}(\{dip\}),\mathtt{U}(m))\ \}$$

which finishes the induction step for the case of the Unicast (T3-1) inference rule.

Unicast (T3-2): AWN defines the inference rule

$$\dfrac{\mathrm{P} \xrightarrow{\neg\mathbf{unicast}(dip,m)} \mathrm{P}' \qquad dip \notin R}{ip:\mathrm{P}:R \xrightarrow{\tau} ip:\mathrm{P}':R}\ \text{Unicast (T3-2)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\tau \in A_1 \wedge ip:\mathrm{P}:R \xrightarrow{A_1} ip:\mathrm{P}':R$, namely Pair 2.3 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip:\mathrm{P}:R) \xrightarrow{a} \mathrm{T}(ip:\mathrm{P}':R)$ for all $a \in A_2 = \{\ \tau\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\neg\mathbf{uni}}(t_{\mathtt{U}(dip)},t_{\mathtt{U}(m)}) \xrightarrow{[\![\overline{\neg\mathbf{uni}}(t_{\mathtt{U}(dip)},t_{\mathtt{U}(m)})]\!]} \checkmark}\ \text{Axiom}}{\overline{\neg\mathbf{uni}}(t_{\mathtt{U}(dip)},t_{\mathtt{U}(m)}) \xrightarrow{\overline{\neg\mathbf{uni}}([\![t_{\mathtt{U}(dip)}]\!],[\![t_{\mathtt{U}(m)}]\!])} \checkmark}\ \text{Definition 4.10}}{\overline{\neg\mathbf{uni}}(t_{\mathtt{U}(dip)},t_{\mathtt{U}(m)}) \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \checkmark}\ \text{Lemma 4.4}}{\overline{\neg\mathbf{uni}}(t_{\mathtt{U}(dip)},t_{\mathtt{U}(m)}).\mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Seq 1}}{[\![t_{\mathtt{U}(dip)} \notin t_{\mathtt{U}(R)}]\!] = \mathit{true}\quad (t_{\mathtt{U}(dip)} \notin t_{\mathtt{U}(R)}) \to \overline{\neg\mathbf{uni}}(t_{\mathtt{U}(dip)},t_{\mathtt{U}(m)}).\mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Guard 2}}{\big(\ (\mathtt{d} \notin t_{\mathtt{U}(R)}) \to \overline{\neg\mathbf{uni}}(\mathtt{D},\mathtt{msg}).\mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})\ \big)\ [\ \mathtt{d} := t_{\mathtt{U}(\{dip\})}, \mathtt{msg} := t_{\mathtt{U}(m)}\ ] \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Substitution}}{\sum_{\mathtt{d}:\mathrm{IP},\mathtt{msg}:\mathrm{MSG}}(\mathtt{d} \notin t_{\mathtt{U}(R)}) \to \overline{\neg\mathbf{uni}}(\mathtt{d},\mathtt{msg}).\mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Sum 2}\quad (2\text{ times})}{\sum_{\mathtt{d}:\mathrm{IP},\mathtt{msg}:\mathrm{MSG}}(\mathtt{d} \notin t_{\mathtt{U}(R)}) \to \overline{\neg\mathbf{uni}}(\mathtt{d},\mathtt{msg}).\mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) + \mathrm{S}[\ \mathtt{ip} := t_{\mathtt{U}(ip)}, \mathtt{R} := t_{\mathtt{U}(R)}\ ] \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Choice 2}}{\Big(\sum_{\mathtt{d}:\mathrm{IP},\mathtt{msg}:\mathrm{MSG}}(\mathtt{d} \notin \mathtt{R}) \to \overline{\neg\mathbf{uni}}(\mathtt{d},\mathtt{msg}).\mathrm{G}(\mathtt{ip},\mathtt{R}) + \mathrm{S}\Big)\ [\ \mathtt{ip} := t_{\mathtt{U}(ip)}, \mathtt{R} := t_{\mathtt{U}(R)}\ ] \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Substitution}}{\text{Definition of G}\quad \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) \xrightarrow{\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Recursion 2}$$

where $\mathrm{S}[\ \mathtt{ip} := t_{\mathtt{U}(ip)}, \mathtt{R} := t_{\mathtt{U}(R)}\ ]$ is an expression equal to all summands of G except for the second one.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\neg\mathbf{uni}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{T}(\mathrm{P}')$ because Pair 2.7 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}\ .\ \neg\mathbf{unicast}(dip,m) \in B_1$. Combining this with the conclusion of the derivation above gives

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\neg\mathbf{uni}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis} \quad \textit{(see above)}}{\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}) \xrightarrow{\neg\mathbf{uni}(\mathtt{U}(dip),\mathtt{U}(m))\mid\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m))} \mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})}\ \text{Par 6}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\gamma_C(\neg\mathbf{uni}(\mathtt{U}(dip),\mathtt{U}(m))\mid\overline{\neg\mathbf{uni}}(\mathtt{U}(dip),\mathtt{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\neg\mathbf{unicast}(\mathtt{U}(dip),\mathtt{U}(m))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet\neg\mathbf{unicast}(\mathtt{U}(dip),\mathtt{U}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{Rename 2}}{\tau\in V\cup\{\tau\}\quad \nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)})) \xrightarrow{\tau} \nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{U}(ip)},t_{\mathtt{U}(R)}))}\ \text{T14}$$

$$\mathrm{T}(ip:\mathrm{P}:R) \xrightarrow{\tau} \mathrm{T}(ip:\mathrm{P}':R)$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R) \text{ for all } a \in A_2 = \{\ \tau\ \}$$

which finishes the induction step for the case of the UNICAST (T3-2) inference rule.

Deliver (T3): AWN defines the inference rule

$$\frac{\mathrm{P} \xrightarrow{\mathbf{deliver}(d)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{ip:\mathbf{deliver}(d)} ip : \mathrm{P}' : R}\ \textsc{Deliver (T3)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $ip : \mathbf{deliver}(d) \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R$, namely Pair 2.12 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R)$ for all $a \in A_2 = \{\ \mathbf{deliver}(\mathrm{U}(ip), [\![\mathrm{T}_\xi(d)]\!])\ \}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\mathbf{del}}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(d)}) \xrightarrow{[\![\overline{\mathbf{del}}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(d)})]\!]} \checkmark}\ \textsc{Axiom}}{\overline{\mathbf{del}}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(d)}) \xrightarrow{\overline{\mathbf{del}}([\![t_{\mathrm{U}(ip)}]\!], [\![t_{\mathrm{U}(d)}]\!])} \checkmark}\ \text{Definition 4.10}}{\overline{\mathbf{del}}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(d)}) \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \checkmark}\ \text{Lemma 4.4}}{\overline{\mathbf{del}}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(d)}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Seq 1}}{\left(\overline{\mathbf{del}}(t_{\mathrm{U}(ip)}, \mathtt{msg}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})\right)[\ \mathtt{data} := t_{\mathrm{U}(d)}\ ] \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\sum_{\mathtt{data:DATA}} \overline{\mathbf{del}}(\mathrm{d}, \mathtt{data}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Sum 2}}{\sum_{\mathtt{data:DATA}} \overline{\mathbf{del}}(t_{\mathrm{U}(ip)}, \mathtt{data}).\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) + \mathrm{S} \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Choice 2}}{\left(\sum_{\mathtt{data:DATA}} \overline{\mathbf{del}}(\mathtt{ip}, \mathtt{data}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S}\right)[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(\{ip\})}\ ] \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Substitution}}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Recursion 2}}\ \text{Definition of G}$$

where $\mathrm{S}[\ \mathtt{ip} := t_{\mathrm{U}(ip)}, \mathtt{R} := t_{\mathrm{U}(R)}\ ]$ is an expression equal to all summands of G except for the third one.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{del}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{T}(\mathrm{P}')$ because Pair 2.9 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{deliver}(d) \in B_1$. Combining this with the conclusion of the derivation above gives

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{del}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis} \quad \dfrac{}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textit{(see above)}}{\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{del}(\mathrm{U}(ip), \mathrm{U}(d)) \mid \overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{T}(\mathrm{P}') \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textsc{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\gamma_C(\mathbf{del}(\mathrm{U}(ip), \mathrm{U}(d)) \mid \overline{\mathbf{del}}(\mathrm{U}(ip), \mathrm{U}(d)))} \Gamma_C(\mathrm{T}(\mathrm{P}') \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{deliver}(\mathrm{U}(ip), \mathrm{U}(d))} \Gamma_C(\mathrm{T}(\mathrm{P}') \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\{\neg \mathbf{unicast} \to \mathbf{t}\} \bullet \mathbf{deliver}(\mathrm{U}(ip), \mathrm{U}(d))} \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Apply } \{\neg \mathbf{unicast} \to \mathbf{t}\}\bullet}{\mathbf{deliver} \in V \cup \{\tau\}} \quad \dfrac{\rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{deliver}(\mathrm{U}(ip), \mathrm{U}(d))} \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}{\nabla_V \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{deliver}(\mathrm{U}(ip), \mathrm{U}(d))} \nabla_V \rho_{\{\neg \mathbf{unicast} \to \mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \textsc{Allow 2}}{\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{\mathbf{deliver}(\mathrm{U}(ip), \mathrm{U}(d))} \mathrm{T}(ip : \mathrm{P}' : R)}\ \textsc{T14}$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R) \text{ for all } a \in A_2 = \{\ \mathbf{deliver}(\mathrm{U}(ip), [\![\mathrm{T}_\xi(d)]\!])\ \}$$

which finishes the induction step for the case of the DELIVER (T3) inference rule.

Internal (T3): AWN defines the inference rule

$$\frac{\mathrm{P} \xrightarrow{\tau} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{\tau} ip : \mathrm{P}' : R}\ \textsc{Internal (T3)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\tau \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R$, namely Pair 2.3 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R)$ for all $a \in A_2 = \{\ \mathbf{t}(\mathtt{u}(D), \mathtt{u}(R), \mathtt{u}(m))\ \}$ for some $D, R : \mathrm{Set(IP)}$ where $R \subseteq D$ and some $m : \mathrm{MSG}$.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T}(\mathrm{P}')$ for some $D, R : \mathrm{Set(IP)}$ where $R \subseteq D$ and some $m : \mathrm{MSG}$ because Pair 2.3 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}\ .\ \tau \in B_1$. This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}\ \textsc{Par 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})) \xrightarrow{\gamma_C(\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m)))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}))}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}))}\ \textsc{Rename 2}}{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}))}\ \text{Apply } \{\neg\mathbf{unicast} \to \mathbf{t}\}\bullet$$

$$\mathbf{t} \trianglelefteq V \cup \{\tau\}\ \dfrac{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}))}{\nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \nabla_V\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \parallel \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}))}\ \textsc{Allow 2}}{\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T}(ip : \mathrm{P}' : R)}\ \textsc{T14}$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R) \text{ for all } a \in A_2 = \{\ \mathbf{t}(\mathtt{u}(D), \mathtt{u}(R), \mathtt{u}(m))\ \}$$

which finishes the induction step for the case of the Internal (T3) inference rule.

Arrive (T3-1): AWN defines the inference rule

$$\dfrac{\mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}'}{ip : \mathrm{P} : R \xrightarrow{\{ip\}\neg\emptyset:\mathbf{arrive}(m)} ip : \mathrm{P}' : R}\ \textsc{Arrive (T3-1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\{ip\}\neg\emptyset : \mathbf{arrive}(m) \in A_1 \wedge ip : \mathrm{P} : R \xrightarrow{A_1} ip : \mathrm{P}' : R$, namely Pair 2.13 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R)$ for all $a \in A_2$ where

$$A_2 = \left\{\ \mathbf{arrive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathtt{u}(m)) \;\middle|\; \begin{array}{c} \hat{\mathtt{D}},\hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)} \\ \hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}} \\ \{ip\} \subseteq \hat{\mathtt{D}}\mathtt{'} \end{array} \right\}$$

Note that $\{ip\} \subseteq \hat{\mathtt{D}}\mathtt{'} \Rightarrow \llbracket t_{\mathtt{u}(ip)} \in \hat{\mathtt{D}}\mathtt{'} \rrbracket = \mathit{true}$.
In mCRL2, the following derivation can be made for all $\hat{\mathtt{D}}$ and $\hat{\mathtt{D}}\mathtt{'}$ such that $\hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}} \wedge t_{\mathtt{u}(ip)} \in \hat{\mathtt{D}}\mathtt{'}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, t_{\mathtt{u}(m)}) \xrightarrow{[\![\overline{\mathbf{receive}}(\hat{\mathtt{D}},\hat{\mathtt{D}}\mathtt{'},t_{\mathtt{u}(m)})]\!]} \checkmark}\ \textsc{Axiom}}{\overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, t_{\mathtt{u}(m)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\llbracket t_{\mathtt{u}(m)}\rrbracket)} \checkmark}\ \text{Definition 4.10}}{\overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, t_{\mathtt{u}(m)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \checkmark}\ \text{Lemma 4.4}}{\overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, t_{\mathtt{u}(m)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \checkmark}\ \textsc{Seq 1}}{\overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, t_{\mathtt{u}(m)}).\mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}\ \textsc{Guard 2}}{\llbracket t_{\mathtt{u}(ip)} \in \hat{\mathtt{D}}\mathtt{'} \rrbracket = \mathit{true}\quad (t_{\mathtt{u}(ip)} \in \hat{\mathtt{D}}\mathtt{'}) \to \overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, t_{\mathtt{u}(m)}).\mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}\ \text{Substitution}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{Definition\ of\ G}}}{\Big(\sum_{\mathtt{D},\mathtt{D}\mathtt{'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(\mathtt{ip} \in \mathtt{D}\mathtt{'}) \to \overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S}\Big)\ [\ \mathtt{ip} := t_{\mathtt{u}(ip)}, \mathtt{R} := t_{\mathtt{u}(R)}\ ] \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}\ \textsc{Recursion 2}}{\mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Big(\sum_{\mathtt{D},\mathtt{D}\mathtt{'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(\mathtt{ip} \in \mathtt{D}\mathtt{'}) \to \overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) + \mathrm{S}\Big)\ [\ \mathtt{ip} := t_{\mathtt{u}(ip)}, \mathtt{R} := t_{\mathtt{u}(R)}\ ] \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}\ \textsc{Sum 2}\ \text{(3 times)}}{\sum_{\mathtt{D},\mathtt{D}\mathtt{'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(t_{\mathtt{u}(ip)} \in \mathtt{D}\mathtt{'}) \to \overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, \mathtt{msg}).\mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}\ \textsc{Choice 2}}{\sum_{\mathtt{D},\mathtt{D}\mathtt{'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}}(t_{\mathtt{u}(ip)} \in \mathtt{D}\mathtt{'}) \to \overline{\mathbf{receive}}(\hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'}, \mathtt{msg}).\mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)}) + \mathrm{S}[\ \mathtt{ip} := t_{\mathtt{u}(ip)}, \mathtt{R} := t_{\mathtt{u}(R)}\ ] \xrightarrow{\overline{\mathbf{receive}}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{G}(t_{\mathtt{u}(ip)}, t_{\mathtt{u}(R)})}\ \text{Substitution}$$

where $\mathrm{S}[\ \mathtt{ip} := t_{\mathtt{u}(ip)}, \mathtt{R} := t_{\mathtt{u}(R)}\ ]$ is an expression equal to all summands of G except for the one containing $\overline{\mathbf{receive}}$.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{receive}(\llbracket\hat{\mathtt{D}}\rrbracket,\llbracket\hat{\mathtt{D}}\mathtt{'}\rrbracket,\mathtt{u}(m))} \mathrm{T}(\mathrm{P}')$ because Pair 2.10 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}\ .\ \mathbf{receive}(m) \in B_1$. Combining this with the conclusion of the derivation above gives

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{receive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis} \qquad \dfrac{}{\mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\overline{\mathbf{receive}}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \textit{(see above)}}{\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}) \xrightarrow{\mathbf{receive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m)) | \overline{\mathbf{receive}}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})}\ \text{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\gamma_C(\mathbf{receive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m)) | \overline{\mathbf{receive}}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Apply } \gamma_C}{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\{\neg\mathbf{unicast}\to\mathbf{t}\}\bullet\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{Rename 2}}{\mathbf{arrive} \subseteq V \cup \{\tau\}} \quad \dfrac{\rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}{\nabla_V \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}) \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \nabla_V \rho_{\{\neg\mathbf{unicast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{P}') \,||\, \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)}))}\ \text{T14}}\ \text{Allow 2}}{\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \mathrm{T}(ip : \mathrm{P}' : R)}\ \text{Apply } \{\neg\mathbf{unicast} \to \mathbf{t}\}\bullet$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R) \text{ for all } a \in A_2 = \left\{ \mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m)) \;\middle|\; \begin{smallmatrix} \hat{\mathrm{D}}, \hat{\mathrm{D}}\text{'}:\mathrm{Set(IP)} \\ \hat{\mathrm{D}}\text{'} \subseteq \hat{\mathrm{D}} \\ \{ip\} \subseteq \hat{\mathrm{D}}\text{'} \end{smallmatrix} \right\}$$

which finishes the induction step for the case of the Arrive (T3-1) inference rule.

Cast (T4-1): AWN defines the inference rule

$$H \subseteq R \wedge K \cap R = \emptyset \quad \dfrac{\mathrm{M} \xrightarrow{R:\mathbf{cast}(m)} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{H\neg K:\mathbf{arrive}(m)} \mathrm{N}'}{\mathrm{M} \,||\, \mathrm{N} \xrightarrow{R:\mathbf{cast}(m)} \mathrm{M}' \,||\, \mathrm{N}'}\ \text{Cast (T4-1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $R : \mathbf{cast}(m) \in A_1 \wedge \mathrm{M} \,||\, \mathrm{N} \xrightarrow{A_1} \mathrm{M}' \,||\, \mathrm{N}'$, namely Pair 2.11 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(\mathrm{M} \,||\, \mathrm{N}) \xrightarrow{a} \mathrm{T}(\mathrm{M}' \,||\, \mathrm{N}')$ for all $a \in \{\ \mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))\ \}$ for some D with $R \subseteq D$.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{M}')$ because Pair 2.11 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $R : \mathbf{cast}(m) \in B_1$.
Similarly, $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \mathrm{T}(\mathrm{N}')$ for all $\hat{\mathrm{D}}\text{'} \subseteq \hat{\mathrm{D}}$ because Pair 2.13 is the only $(B_1, B_2) \in \mathcal{A}$. $H\neg K : \mathbf{arrive}(m) \in B_1$. This observation about $\mathrm{T}(\mathrm{M})$ can be refined as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}\text{'} \rrbracket, \mathrm{U}(m))} \mathrm{T}(\mathrm{N}')}\ \text{Induction hypothesis}}{H \subseteq R \wedge K \cap R = \emptyset}\ \text{Choose } \hat{\mathrm{D}}\text{'} = t_{\mathrm{U}(R)}}{R \subseteq D}\ \text{Choose } \hat{\mathrm{D}} = t_{\mathrm{U}(D)}}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\llbracket t_{\mathrm{U}(D)} \rrbracket, \llbracket t_{\mathrm{U}(R)} \rrbracket, \mathrm{U}(m))} \mathrm{T}(\mathrm{N}')}\ \text{Lemma 4.4 (2 times)}}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{N}')}$$

This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{M}')}\ \text{Induction hypothesis} \qquad \dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{N}')}\ \textit{(see above)}}{\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m)) | \mathbf{arrive}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}')}\ \text{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_C(\mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m)) | \mathbf{arrive}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}'))}\ \text{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{s}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}'))}\ \text{Apply } \gamma_C}{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}(\mathbf{s}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m)))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}'))}\ \text{Comm 2}}{\mathbf{s} \subseteq V \cup \{\tau\}} \quad \dfrac{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{s}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}'))}{\nabla_V \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{s}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \nabla_V \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}'))}\ \text{Allow 2}}\ \text{Apply } \gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}}{\rho_R \nabla_V \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{R\bullet\mathbf{s}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \rho_R \nabla_V \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}'))}\ \text{Rename 2}}{\rho_R \nabla_V \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \rho_R \nabla_V \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}'))}\ \text{Apply } R\bullet$$
$$\mathrm{T}(\mathrm{M} \,||\, \mathrm{N}) \xrightarrow{\mathbf{starcast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))} \mathrm{T}(\mathrm{M}' \,||\, \mathrm{N}')\ \text{T15}$$

Clearly, Pair 2.11 applies to the conclusion of this derivation, and thus the induction step of Lemma 4.6 for the case of Cast (T4-1) is established.

Cast (T4-2): Similar to the Lemma 4.6-proof for Cast (T4-1).

Cast (T4-3): AWN defines the inference rule

$$\frac{\mathrm{M} \xrightarrow{H\neg K:\mathbf{arrive}(m)} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{H'\neg K':\mathbf{arrive}(m)} \mathrm{N}'}{\mathrm{M} \mid\mid \mathrm{N} \xrightarrow{(H\cup H')\neg(K\cup K'):\mathbf{arrive}(m)} \mathrm{M}' \mid\mid \mathrm{N}'}\ \textsc{Cast (T4-3)}$$

There exist an (A_1, A_2) in $\mathcal{A}$ such that $(H \cup H')\neg(K \cup K') : \mathbf{arrive}(m) \in A_1 \wedge \mathrm{M} \mid\mid \mathrm{N} \xrightarrow{A_1} \mathrm{M}' \mid\mid \mathrm{N}'$, namely Pair 2.13 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) \xrightarrow{a} \mathrm{T}(\mathrm{M}' \mid\mid \mathrm{N}')$ for all $a \in A_2$ where

$$A_2 = \left\{ \mathbf{arrive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathtt{U}(m)) \;\middle|\; \begin{array}{c} \hat{\mathtt{D}},\hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)} \\ \hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}} \\ H \cup H' \subseteq \hat{\mathtt{D}}\mathtt{'} \\ (K \cup K') \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset \end{array} \right\}$$

From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathtt{U}(m))} \mathrm{T}(\mathrm{M}')$ for all $\hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}}$ because Pair 2.13 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $H\neg K : \mathbf{arrive}(m) \in B_1$. For the same reason, $\mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathtt{U}(m))} \mathrm{T}(\mathrm{N}')$ for all $\hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}}$. This means that the following derivation can be made in mCRL2 for $R \subseteq D$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathtt{U}(m))} \mathrm{T}(\mathrm{M}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \mathrm{T}(\mathrm{M}')}\ \text{Choose } \llbracket \hat{\mathtt{D}} \rrbracket = \mathtt{U}(D),\ \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket = \mathtt{U}(R) \qquad \dfrac{\dfrac{}{\mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{arrive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathtt{U}(m))} \mathrm{T}(\mathrm{N}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \mathrm{T}(\mathrm{N}')}\ \text{Choose } \llbracket \hat{\mathtt{D}} \rrbracket = \mathtt{U}(D),\ \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket = \mathtt{U}(R)}{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)) \mid \mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}')}\ \textsc{Par 3}$$

$$\dfrac{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)) \mid \mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}')}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_C(\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)) \mid \mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\ \text{Apply } \gamma_C$$

$$\dfrac{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_C(\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)) \mid \mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}{\Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}(\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)) \mid \mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m)))} \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\ \text{Apply } \gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}$$

$$\mathbf{a} \in V \cup \{\tau\}\ \dfrac{\Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{a}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}{\nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{a}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\ \textsc{Allow 2}$$

$$\dfrac{\nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{a}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}{\rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{R \bullet \mathbf{a}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\ \textsc{Rename 2}$$

$$\dfrac{\rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{R \bullet \mathbf{a}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}{\rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\ \text{Apply } R\bullet$$

$$\dfrac{\rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \rho_R \nabla_V \Gamma_{\{\mathbf{arrive} \mid \mathbf{arrive} \to \mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}{\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) \xrightarrow{\mathbf{arrive}(\mathtt{U}(D), \mathtt{U}(R), \mathtt{U}(m))} \mathrm{T}(\mathrm{M}' \mid\mid \mathrm{N}')}\ \text{T15}$$

where $\hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}}$, $H \subseteq \hat{\mathtt{D}}\mathtt{'}$, $H' \subseteq \hat{\mathtt{D}}\mathtt{'}$, $K \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset$ and $K' \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset$ (according to the induction hypothesis). The side condition of Pair 2.13 holds for the conclusion of the derivation above:

$$\hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}} \Leftrightarrow \hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}}$$
$$H \subseteq \hat{\mathtt{D}}\mathtt{'} \wedge H' \subseteq \hat{\mathtt{D}}\mathtt{'} \Leftrightarrow (H \cup H') \subseteq \hat{\mathtt{D}}\mathtt{'}$$
$$K \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset \wedge K' \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset \Leftrightarrow (K \cup K') \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset$$

This proves that

$$\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) \xrightarrow{a} \mathrm{T}(\mathrm{M}' \mid\mid \mathrm{N}') \text{ for all } a \in \left\{ \mathbf{arrive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathtt{U}(m)) \;\middle|\; \begin{array}{c} \hat{\mathtt{D}},\hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)} \\ \hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}} \\ H \cup H' \subseteq \hat{\mathtt{D}}\mathtt{'} \\ (K \cup K') \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset \end{array} \right\}$$

finishing the induction step for the case of the Cast (T4-3) inference rule.

Cast (T4-4): AWN defines the inference rule

$$\frac{\mathrm{M} \xrightarrow{R:*\mathbf{cast}(m)} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\tau} [\ \mathrm{M}'\ ]}\ \textsc{Cast (T4-4)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\tau \in A_1 \wedge [\ \mathrm{M}\ ] \xrightarrow{A_1} [\ \mathrm{M}'\ ]$, namely Pair 2.3 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{a} \mathrm{T}([\ \mathrm{M}'\ ])$ for all $a \in A_2 = \{\ \mathbf{t}(\mathsf{u}(D), \mathsf{u}(R), \mathsf{u}(m))\ \}$ for some $D, R : \mathrm{Set}(\mathrm{IP})$ where $R \subseteq D$ and some $m : \mathrm{MSG}$.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{starcast}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \mathrm{T}(\mathrm{M}')$ for some $D \supseteq R$ because Pair 2.11 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}\ .\ R : \mathbf{*cast}(m) \in B_1$. This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{starcast}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \mathrm{T}(\mathrm{M}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H} \xrightarrow{\mathbf{starcast}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \mathrm{T}(\mathrm{M}') \,||\, \mathrm{H}}\ \textsc{Par 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\gamma_C(\mathbf{starcast}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\mathbf{starcast}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{G}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet\mathbf{starcast}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{G})}\ \textsc{Rename 2}}{\mathbf{t} \in V \cup \{\tau\}\quad \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{G}) \xrightarrow{\mathbf{t}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{G})}\ \text{Apply } \{\mathbf{starcast}\to\mathbf{t}\}\bullet}{\nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{G}) \xrightarrow{\mathbf{t}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{G})}\ \textsc{Allow 2}}{\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{\mathbf{t}(\mathsf{u}(D),\mathsf{u}(R),\mathsf{u}(m))} \mathrm{T}([\ \mathrm{M}'\ ])}\ \textsc{T16}$$

So it is indeed the case that

$$\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{a} \mathrm{T}([\ \mathrm{M}'\ ]) \text{ for all } a \in A_2 = \{\ \mathbf{t}(\mathsf{u}(D), \mathsf{u}(R), \mathsf{u}(m))\ \}$$

which finishes the induction step for the case of the CAST (T4-4) inference rule.

Deliver (T4-1): AWN defines the inference rule

$$\dfrac{\mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}'}{\mathrm{M} \,||\, \mathrm{N} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}' \,||\, \mathrm{N}}\ \textsc{Deliver (T4-1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $ip : \mathbf{deliver}(d) \in A_1 \wedge \mathrm{M} \,||\, \mathrm{N} \xrightarrow{A_1} \mathrm{M}' \,||\, \mathrm{N}$, namely Pair 2.12 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}(\mathrm{M} \,||\, \mathrm{N}) \xrightarrow{a} \mathrm{T}(\mathrm{M}' \,||\, \mathrm{N})$ for all $a \in A_2 = \{\ \mathbf{deliver}(\mathsf{u}(ip), \mathsf{u}(d))\ \}$.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \mathrm{T}(\mathrm{M}')$ because Pair 2.12 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}\ .\ ip : \mathbf{deliver}(d) \in B_1$. This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \mathrm{T}(\mathrm{M}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N})}\ \textsc{Par 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_C(\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}))}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}))}\ \text{Apply } \gamma_C}{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}(\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d)))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}))}\ \textsc{Comm 2}}{\mathbf{deliver} \in V \cup \{\tau\}\quad \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}))}\ \text{Apply } \gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}}{\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}))}\ \textsc{Allow 2}}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{R\bullet\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}))}\ \textsc{Rename 2}}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{T}(\mathrm{N})) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{T}(\mathrm{N}))}\ \text{Apply } R\bullet}{\mathrm{T}(\mathrm{M} \,||\, \mathrm{N}) \xrightarrow{\mathbf{deliver}(\mathsf{u}(ip),\mathsf{u}(d))} \mathrm{T}(\mathrm{M}' \,||\, \mathrm{N})}\ \textsc{T15}$$

So it is indeed the case that

$$\mathrm{T}(\mathrm{M} \,||\, \mathrm{N}) \xrightarrow{a} \mathrm{T}(\mathrm{M}' \,||\, \mathrm{N}) \text{ for all } a \in A_2 = \{\ \mathbf{deliver}(\mathsf{u}(ip), \mathsf{u}(d))\ \}$$

which finishes the induction step for the case of the DELIVER (T4-1) inference rule.

Deliver (T4-2): Similar to the Lemma 4.6-proof for Deliver (T4-1).

Deliver (T4-3): AWN defines the inference rule

$$\frac{\mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}'}{[\,\mathrm{M}\,] \xrightarrow{ip:\mathbf{deliver}(d)} [\,\mathrm{M}'\,]}\ \textsc{Deliver (T4-3)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $ip : \mathbf{deliver}(d) \in A_1 \wedge \mathrm{M} \mid\mid \mathrm{N} \xrightarrow{A_1} \mathrm{M}' \mid\mid \mathrm{N}$, namely Pair 2.12 in Table 2.6. The induction step can therefore be proven for this particular case by finding a set of derivations in mCRL2 for $\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{a} \mathrm{T}([\,\mathrm{M}'\,])$ for all $a \in A_2 = \{\, \mathbf{deliver}(\mathrm{u}(ip), \mathrm{u}(d)) \,\}$.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \mathrm{T}(\mathrm{M}')$ because Pair 2.12 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $ip : \mathbf{deliver}(d) \in B_1$. This means that the following derivation can be made in mCRL2:

$$\mathbf{deliver} \in V \cup \{\tau\}\ \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \mathrm{T}(\mathrm{M}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H} \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H}}\ \textsc{Par 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \textsc{Rename 2}}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \text{Apply } \{\mathbf{starcast} \to \mathbf{t}\}\bullet}{\nabla_V \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \nabla_V \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \textsc{Allow 2}}{\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{\mathbf{deliver}(\mathrm{u}(ip),\mathrm{u}(d))} \mathrm{T}([\,\mathrm{M}'\,])}\ \textsc{T16}$$

So it is indeed the case that

$$\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{a} \mathrm{T}([\,\mathrm{M}'\,]) \text{ for all } a \in A_2 = \{\, \mathbf{deliver}(\mathrm{u}(ip), \mathrm{u}(d)) \,\}$$

which finishes the induction step for the case of the Deliver (T4-3) inference rule.

Internal (T4-1): Similar to the Lemma 4.6-proof for Deliver (T4-1).

Internal (T4-2): Similar to the Lemma 4.6-proof for Deliver (T4-1).

Internal (T4-3): Similar to the Lemma 4.6-proof for Deliver (T4-3).

Connect (T4-1): AWN defines the inference rule

$$\frac{\mathrm{M} \xrightarrow{\mathbf{connect}(ip,ip')} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{\mathbf{connect}(ip,ip')} \mathrm{N}'}{\mathrm{M} \mid\mid \mathrm{N} \xrightarrow{\mathbf{connect}(ip,ip')} \mathrm{M}' \mid\mid \mathrm{N}'}\ \textsc{Connect (T4-1)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{connect}(ip, ip') \in A_1 \wedge \mathrm{M} \mid\mid \mathrm{N} \xrightarrow{A_1} \mathrm{M}' \mid\mid \mathrm{N}'$, namely Pair 2.14 in Table 2.6. For this particular case, the induction step can be proven by finding a set of derivations in mCRL2 for $\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) \xrightarrow{A_1} \mathrm{T}(\mathrm{M}' \mid\mid \mathrm{N}')$ for all $a \in A_2 = \{\, \mathbf{connect}(\mathrm{u}(ip), \mathrm{u}(ip')) \,\}$.
From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{connect}(\mathrm{u}(ip),\mathrm{u}(ip'))} \mathrm{T}(\mathrm{M}')$ because Pair 2.14 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{connect}(ip, ip') \in B_1$. For the same reason, $\mathrm{T}(\mathrm{N}) \xrightarrow{\mathbf{connect}(\mathrm{u}(ip),\mathrm{u}(ip'))} \mathrm{T}(\mathrm{N}')$. This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T(M)} \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T(M')}}\text{ Induction hypothesis} \quad \dfrac{}{\mathrm{T(N)} \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T(N')}}\text{ Induction hypothesis}}{\mathrm{T(M)} \mid\mid \mathrm{T(N)} \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))\mid\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T(M')} \mid\mid \mathrm{T(N')}}\text{ Par 3}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\gamma_C(\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))\mid\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip')))} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N')})}\text{ Comm 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{c}(\mathrm{U}(ip),\mathrm{U}(ip'))} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N')})}\text{ Apply } \gamma_C}{\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}(\mathbf{c}(\mathrm{U}(ip),\mathrm{U}(ip')))} \Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N')})}\text{ Comm 2}}{\mathbf{c}\in V \cup \{\tau\}\quad \dfrac{\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{c}(\mathrm{U}(ip),\mathrm{U}(ip'))} \Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N')})}{\nabla_V\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{c}(\mathrm{U}(ip),\mathrm{U}(ip'))} \nabla_V\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N')})}\text{ Allow 2}}\text{ Apply } \gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{R\bullet\mathbf{c}(\mathrm{U}(ip),\mathrm{U}(ip'))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N')})}\text{ Rename 2}}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}\mid\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N')})}\text{ Apply } R\bullet}{\mathrm{T(M} \mid\mid \mathrm{N)} \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T(M'} \mid\mid \mathrm{N')}}\text{ T15}$$

So it is indeed the case that

$$\mathrm{T(M} \mid\mid \mathrm{N)} \xrightarrow{a} \mathrm{T(M'} \mid\mid \mathrm{N) \text{ for all } } a \in A_2 = \{\ \mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))\ \}$$

which finishes the induction step for the case of the Connect (T4-1) inference rule.

Connect (T4-2): AWN defines the inference rule

$$\dfrac{\mathrm{M} \xrightarrow{\mathbf{connect}(ip,ip')} \mathrm{M'}}{[\ \mathrm{M}\ ] \xrightarrow{\mathbf{connect}(ip,ip')} [\ \mathrm{M'}\ ]}\ \text{Connect (T4-2)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $\mathbf{connect}(ip, ip') \in A_1 \wedge [\ \mathrm{M}\ ] \xrightarrow{A_1} [\ \mathrm{M'}\ ]$, namely Pair 2.14 in Table 2.6. For this particular case, the induction step can be proven by finding a set of derivations in mCRL2 for $\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{A_1} \mathrm{T}([\ \mathrm{M'}\ ])$ for all $a \in A_2$ where $A_2 = \{\ \mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))\ \}$.
From the induction hypothesis, it follows that $\mathrm{T(M)} \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T(M')}$ because Pair 2.14 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{connect}(ip, ip') \in B_1$. This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T(M)} \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T(M')}}\text{ Induction hypothesis}}{\mathrm{T(M)} \mid\mid \mathrm{H} \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T(M')} \mid\mid \mathrm{H}}\text{ Par 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\gamma_C(\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip')))} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Comm 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Rename 2}}{\mathbf{connect}\in V \cup \{\tau\}\quad \dfrac{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}{\nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Allow 2}}\text{ Apply } \{\mathbf{starcast}\to\mathbf{t}\}\bullet}{\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{\mathbf{connect}(\mathrm{U}(ip),\mathrm{U}(ip'))} \mathrm{T}([\ \mathrm{M'}\ ])}\text{ T16}$$

So it is indeed the case that

$$\mathrm{T(M} \mid\mid \mathrm{N)} \xrightarrow{a} \mathrm{T(M'} \mid\mid \mathrm{N) \text{ for all } } a \in A_2 = \{\ \mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip'))\ \}$$

which finishes the induction step for the case of the Connect (T4-2) inference rule.

Disconnect (T4-1): Similar to the Lemma 4.6-proof for Connect (T4-1).

Disconnect (T4-2): Similar to the Lemma 4.6-proof for Connect (T4-2).

Newpkt (T4): AWN defines the inference rule

$$\frac{\mathrm{M} \xrightarrow{\{ip\}\neg K:\mathbf{arrive}(\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)}))} \mathrm{M}'}{[\,\mathrm{M}\,] \xrightarrow{ip:\mathbf{newpkt}(d,dip)} [\,\mathrm{M}'\,]}\ \textsc{Newpkt (T4)}$$

There exists an (A_1, A_2) in $\mathcal{A}$ such that $ip : \mathbf{newpkt}(d, dip) \in A_1 \wedge [\,\mathrm{M}\,] \xrightarrow{A_1} [\,\mathrm{M}'\,]$, namely Pair 2.16 in Table 2.6. This base case can therefore be proven by finding a set of derivations in mCRL2 such that $\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{a} \mathrm{T}([\,\mathrm{M}'\,])$ for all $a \in A_2 = \{\, \mathbf{newpkt}(\{\mathtt{U}(ip)\}, \{\mathtt{U}(ip)\}, \mathtt{newpkt}(\mathtt{U}(d), \mathtt{U}(dip)))\,\}$.
In mCRL2, the following derivation can be made:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\overline{\mathbf{newpkt}}(\{t_{\mathtt{U}(ip)}\},\{t_{\mathtt{U}(ip)}\},\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)})) \xrightarrow{[\![\overline{\mathbf{newpkt}}(\{t_{\mathtt{U}(ip)}\},\{t_{\mathtt{U}(ip)}\},\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)}))]\!]} \checkmark}\textsc{Axiom}}{\overline{\mathbf{newpkt}}(\{t_{\mathtt{U}(ip)}\},\{t_{\mathtt{U}(ip)}\},\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)})) \xrightarrow{\overline{\mathbf{newpkt}}(\{[\![t_{\mathtt{U}(ip)}]\!]\},\{[\![t_{\mathtt{U}(ip)}]\!]\},\mathtt{newpkt}([\![t_{\mathtt{U}(d)}]\!],[\![t_{\mathtt{U}(dip)}]\!]))} \checkmark}\text{Definition 4.10}}{\overline{\mathbf{newpkt}}(\{t_{\mathtt{U}(ip)}\},\{t_{\mathtt{U}(ip)}\},\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)})) \xrightarrow{\overline{\mathbf{newpkt}}(\{[\![t_{\mathtt{U}(ip)}]\!]\},\{[\![t_{\mathtt{U}(ip)}]\!]\},\mathtt{newpkt}([\![t_{\mathtt{U}(d)}]\!],[\![t_{\mathtt{U}(dip)}]\!]))} \checkmark}\text{Lemma 4.4}}{\overline{\mathbf{newpkt}}(\{t_{\mathtt{U}(ip)}\},\{t_{\mathtt{U}(ip)}\},\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)})) \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \checkmark}\textsc{Seq 2}}{\overline{\mathbf{newpkt}}(\{t_{\mathtt{U}(ip)}\},\{t_{\mathtt{U}(ip)}\},\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)})).\mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{H}}\text{Substitution}}{(\overline{\mathbf{newpkt}}(\{\mathtt{ip}\},\{\mathtt{ip}\},\mathtt{newpkt}(\mathtt{data},\mathtt{dest})).\mathrm{H})\,[\mathtt{ip} := t_{\mathtt{U}(ip)}, \mathtt{data} := t_{\mathtt{U}(d)}, \mathtt{dest} := t_{\mathtt{U}(dip)}] \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{H}}\textsc{Sum 2}}{\sum_{\mathtt{ip:IP},\mathtt{data:DATA},\mathtt{dest:IP}} \overline{\mathbf{newpkt}}(\{\mathtt{ip}\},\{\mathtt{ip}\},\mathtt{newpkt}(\mathtt{data},\mathtt{dest})).\mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{H}}\text{Substitution}}{\text{Definition of H}\ \dfrac{(\sum_{\mathtt{ip:IP},\mathtt{data:DATA},\mathtt{dest:IP}} \overline{\mathbf{newpkt}}(\{\mathtt{ip}\},\{\mathtt{ip}\},\mathtt{newpkt}(\mathtt{data},\mathtt{dest})).\mathrm{H})[\,] \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{H}}{\mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{H}}\textsc{Recursion 2}}$$

From the induction hypothesis, it follows that $\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\mathtt{'}]\!],\mathtt{newpkt}(t_{\mathtt{U}(d)},t_{\mathtt{U}(dip)}))} \mathrm{T}(\mathrm{M}')$ for all $\hat{\mathtt{D}}\mathtt{'} \subseteq \hat{\mathtt{D}}$ such that $\{ip\} \subseteq \hat{\mathtt{D}}\mathtt{'} \wedge K \cap \hat{\mathtt{D}}\mathtt{'} = \emptyset$ because Pair 2.13 in Table 2.6 is the only $(B_1, B_2) \in \mathcal{A}$. $\mathbf{arrive}(\mathtt{newpkt}(t_{\mathtt{U}(d)}, t_{\mathtt{U}(dip)})) \in B_1$. This observation about $\mathrm{T}(\mathrm{M})$ can be refined as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\mathtt{'}]\!],\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{T}(\mathrm{M}')}\text{Induction hypothesis}}{\{ip\} \subseteq R \wedge K \cap R = \emptyset}\ \text{Choose } \hat{\mathtt{D}} = \hat{\mathtt{D}}\mathtt{'} = \{t_{\mathtt{U}(ip)}\}}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}([\![\{t_{\mathtt{U}(ip)}\}]\!],[\![\{t_{\mathtt{U}(ip)}\}]\!],\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{T}(\mathrm{M}')}}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\{[\![t_{\mathtt{U}(ip)}]\!]\},\{[\![t_{\mathtt{U}(ip)}]\!]\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{T}(\mathrm{M}')}\text{Lemma 4.4 (2 times)}}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{T}(\mathrm{M}')}$$

This means that the following derivation can be made in mCRL2:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{H}}\textit{(see above)} \quad \dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{arrive}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{T}(\mathrm{M}')}\textit{(see above)}}{\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))|\mathbf{arrive}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{T}(\mathrm{M}') \,||\, \mathrm{H}}\textsc{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\gamma_C(\overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))|\mathbf{arrive}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip))))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\mathbf{newpkt}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet\mathbf{newpkt}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\textsc{Rename 2}}{\mathbf{newpkt} \in V \cup \{\tau\}\quad \dfrac{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\mathbf{newpkt}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}{\nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \,||\, \mathrm{H}) \xrightarrow{\mathbf{newpkt}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \,||\, \mathrm{H})}\textsc{Allow 2}}{\mathrm{T}([\,\mathrm{M}\,]) \xrightarrow{\mathbf{newpkt}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))} \mathrm{T}([\,\mathrm{M}'\,])}\text{T16}$$

So it is indeed the case that

$$\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{T}(ip : \mathrm{P}' : R) \text{ for all } a \in A_2 = \{\, \mathbf{newpkt}(\{\mathtt{U}(ip)\}, \{\mathtt{U}(ip)\}, \mathtt{newpkt}(\mathtt{U}(d), \mathtt{U}(dip)))\,\}$$

which finishes the induction step for the case of the Newpkt (T4) inference rule.

Appendix B

Complete proof of Lemma 4.7

1 Base cases

Translation rule T1: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \textbf{broadcast}(ms).\mathrm{p}) = \textstyle\sum_{\mathtt{D}:\mathrm{Set(IP)}} \textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{D} \notin \textsc{vars}(\mathrm{T}_\xi(ms)) \cup \textsc{vars}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \mathrm{T1}$$

Consider the following derivation for expressions of the form $\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \textbf{broadcast}(ms).\mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{[\![\textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms))]\!]} \checkmark}\ \textsc{Axiom}}{\textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)) \xrightarrow{\textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \checkmark}\ \text{Definition 4.10}}{\textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \hat{\mathtt{D}}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{p})}\ \textsc{Seq}\ 1}{(\textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{p}))[\mathtt{D} := \hat{\mathtt{D}}] \xrightarrow{\textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{p})}\ \text{Substitution}}{\sum_{\mathtt{D}:\mathrm{Set(IP)}} \textbf{cast}(\mathcal{U}_{\mathrm{IP}}, \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{p}) \xrightarrow{\textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{p})}\ \textsc{Sum}\ 2}{\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \textbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\textsc{dom}(\xi)}(\xi, \mathrm{p})}\ \mathrm{T1}}{\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \textbf{broadcast}(ms).\mathrm{p}) \xrightarrow{\textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})}\ \text{Define } \sigma := \emptyset$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that $a \in \left\{ \textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!]) \mid \hat{\mathtt{D}} : \mathrm{Set(IP)} \right\}$ and $\mathrm{Q} = \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$. There is exactly one pair $(A_1, A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\textsc{dom}(\xi)}(\xi, \textbf{broadcast}(ms).\mathrm{p}) \xrightarrow{A_1} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$:

$$(\left\{ \textbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!]) \mid \hat{\mathtt{D}} : \mathrm{Set(IP)} \right\}, \{ \textbf{broadcast}(\xi(ms)) \})$$

This pair satisfies the condition that $\xi, \textbf{broadcast}(ms).\mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule Broadcast (T1)), which is sufficient to prove this particular base case.

Translation rule T2: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \textbf{groupcast}(\mathit{dests}, ms).\mathrm{p}) = \textstyle\sum_{\mathtt{D}:\mathrm{Set(IP)}} \textbf{cast}(\mathrm{T}_\xi(\mathit{dests}), \mathtt{D}, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{D} \notin \textsc{vars}(\mathrm{T}_\xi(\mathit{dests})) \cup \textsc{vars}(\mathrm{T}_\xi(ms)) \cup \textsc{vars}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \mathrm{T2}$$

Consider the following derivation for expressions of the form $\mathrm{T}_{\textsc{dom}(\xi)}(\xi, \textbf{groupcast}(\mathit{dests}, ms).\mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{cast}(\mathrm{T}_\xi(\mathit{dests}),\hat{\mathrm{D}},\mathrm{T}_\xi(\mathit{ms}))\xrightarrow{[\![\mathbf{cast}(\mathrm{T}_\xi(\mathit{dests}),\hat{\mathrm{D}},\mathrm{T}_\xi(\mathit{ms}))]\!]}\checkmark}\,\text{Axiom}}{\mathbf{cast}(\mathrm{T}_\xi(\mathit{dests}),\hat{\mathrm{D}},\mathrm{T}_\xi(\mathit{ms}))\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\checkmark}\,\text{Definition 4.10}}{\mathbf{cast}(\mathrm{T}_\xi(\mathit{dests}),\hat{\mathrm{D}},\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Seq 1}}{(\mathbf{cast}(\mathrm{T}_\xi(\mathit{dests}),\hat{\mathrm{D}},\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}))\,[\mathtt{D}:=\mathrm{T}_\xi(\mathit{dests}),\mathtt{D'}:=\hat{\mathrm{D}}]\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Substitution}}{\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)}}\mathbf{cast}(\mathtt{D},\mathtt{D'},\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Sum 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{groupcast}(\mathit{dests},\mathit{ms}).\mathrm{p})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{T2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{groupcast}(\mathit{dests},\mathit{ms}).\mathrm{p})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})}\,\text{Define }\sigma:=\emptyset$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that $a \in \{\, \mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!]) \mid \hat{\mathrm{D}} : \mathrm{Set(IP)} \,\}$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})$. There is exactly one pair $(A_1,A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{groupcast}(\mathit{dests},\mathit{ms}).\mathrm{p}) \xrightarrow{A_1} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})$:

$$(\{\, \mathbf{cast}([\![\mathrm{T}_\xi(\mathit{dests})]\!],[\![\hat{\mathrm{D}}]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!]) \mid \hat{\mathrm{D}} : \mathrm{Set(IP)} \,\}, \{\, \mathbf{groupcast}(\xi(\mathit{dests}),\xi(\mathit{ms})) \,\})$$

This pair satisfies the condition that $\xi,\mathbf{groupcast}(\mathit{dests},\mathit{ms}).\mathrm{p} \xrightarrow{a'} \xi[\sigma],\mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule Groupcast (T1)), which is sufficient to prove this particular base case.

Translation rule T3: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q}) = \mathbf{cast}(\{\mathrm{T}_\xi(\mathit{dest})\},\{\mathrm{T}_\xi(\mathit{dest})\},\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_V(\xi,\mathrm{p}) + \neg\mathbf{uni}(\{\mathrm{T}_\xi(\mathit{dest})\},\emptyset,\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_V(\xi,\mathrm{q}) \quad \text{T3}$$

Consider the following derivation for expressions of the form $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{cast}(\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\mathit{ms}))\xrightarrow{[\![\mathbf{cast}(\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\mathit{ms}))]\!]}\checkmark}\,\text{Axiom}}{\mathbf{cast}(\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\mathit{ms}))\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\checkmark}\,\text{Definition 4.10}}{\mathbf{cast}(\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Seq 1}}{\mathbf{cast}(\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}) + \neg\mathbf{uni}(\mathrm{T}_\xi(\mathit{dest}),\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{Choice 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p})}\,\text{T3}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q})\xrightarrow{\mathbf{cast}([\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})}\,\text{Define }\sigma:=\emptyset$$

This representative derivation has one alternative where $a \in \{\, \neg\mathbf{uni}([\![\mathrm{T}_\xi(\mathit{dest})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!]) \,\}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg\mathbf{uni}(\mathrm{T}_\xi(\mathit{dest}),\mathrm{T}_\xi(\mathit{ms}))\xrightarrow{[\![\neg\mathbf{uni}(\mathrm{T}_\xi(\mathit{dest}),\mathrm{T}_\xi(\mathit{ms}))]\!]}\checkmark}\,\text{Axiom}}{\neg\mathbf{uni}(\mathrm{T}_\xi(\mathit{dest}),\mathrm{T}_\xi(\mathit{ms}))\xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\mathit{dest})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\checkmark}\,\text{Definition 4.10}}{\neg\mathbf{uni}(\mathrm{T}_\xi(\mathit{dest}),\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})\xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\mathit{dest})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})}\,\text{Seq 1}}{\mathbf{cast}(\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\{\mathit{dest}\}),\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{p}) + \neg\mathbf{uni}(\mathrm{T}_\xi(\mathit{dest}),\mathrm{T}_\xi(\mathit{ms})).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})\xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\mathit{dest})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})}\,\text{Choice 4}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q})\xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\mathit{dest})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathrm{q})}\,\text{T3}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q})\xrightarrow{\neg\mathbf{uni}([\![\mathrm{T}_\xi(\mathit{dest})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!])}\mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{q})}\,\text{Define }\sigma:=\emptyset$$

These derivations are the only representative derivations (see Definition 4.9) for an expression of the form $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{a} \mathrm{Q}$. It follows that there are exactly two cases that must be distinguished:

- Let $a \in \{\, \mathbf{cast}([\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!]) \,\}$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})$. There is one pair $(A_1,A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{A_1} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})$:

  $$(\{\, \mathbf{cast}([\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\{\mathit{dest}\})]\!],[\![\mathrm{T}_\xi(\mathit{ms})]\!]) \,\}, \{\, \mathbf{unicast}(\xi(\mathit{dest}),\xi(\mathit{ms})) \,\})$$

  This pair satisfies the condition that $\xi,\mathbf{unicast}(\mathit{dest},\mathit{ms}).\mathrm{p} \blacktriangleright \mathrm{q} \xrightarrow{a'} \xi[\sigma],\mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule Unicast (T1-1)).

- Let $a \in \{\, \neg\mathbf{uni}([\![\mathrm{T}_\xi(dest)]\!], [\![\mathrm{T}_\xi(ms)]\!]) \,\}$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{q})$. There is one pair $(A_1, A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{unicast}(dest, ms).\mathrm{p} \blacktriangleright \mathrm{q}) \xrightarrow{A_1} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{q})$:

  $$(\{\, \neg\mathbf{uni}([\![\mathrm{T}_\xi(dest)]\!], [\![\mathrm{T}_\xi(ms)]\!]) \,\}, \{\, \neg\mathbf{unicast}(\xi(dest), \xi(ms)) \,\})$$

  This pair satisfies the condition that $\xi, \mathbf{unicast}(dest, ms).\mathrm{p} \blacktriangleright \mathrm{q} \xrightarrow{a'} \xi[\sigma], \mathrm{q}$ can be derived for all $a' \in A_2$ (via AWN inference rule UNICAST (T1-2)).

By proving both cases this particular base case has been established.

Translation rule T4: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \mathbf{send}(\mathrm{T}_\xi(ms)).\mathrm{p}) = \mathbf{send}(\emptyset, \emptyset, \mathrm{T}_\xi(ms)).\mathrm{T}_V(\xi, \mathrm{p}) \quad \text{T4}$$

Consider the following derivation for expressions of the form $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{send}(\mathrm{T}_\xi(ms)).\mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{send}(\emptyset, \emptyset, \mathrm{T}_\xi(dest)) \xrightarrow{[\![\mathbf{send}(\emptyset, \emptyset, \mathrm{T}_\xi(ms))]\!]} \checkmark}\,\text{Axiom}}{\mathbf{send}(\emptyset, \emptyset, \mathrm{T}_\xi(dest)) \xrightarrow{\mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{send}(\emptyset, \emptyset, \mathrm{T}_\xi(dest)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{\mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\,\text{Seq 1}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{send}(ms).\mathrm{p}) \xrightarrow{\mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\,\text{T4}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{send}(ms).\mathrm{p}) \xrightarrow{\mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})}\,\text{Define } \sigma := \emptyset$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that $a \in \{\, \mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!]) \,\}$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$. There is one pair $(A_1, A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{send}(\mathrm{T}_\xi(ms)).\mathrm{p}) \xrightarrow{A_1} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$:

$$(\{\, \mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!]) \,\}, \{\, \mathbf{send}(\xi(ms)) \,\})$$

This pair satisfies the condition that $\xi, \mathbf{send}(\mathrm{T}_\xi(ms)).\mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule SEND (T1)), which is sufficient to prove this particular base case.

Translation rule T5: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \mathbf{deliver}(data).\mathrm{p}) = \textstyle\sum_{\mathtt{ip}:\mathrm{IP}} \mathbf{del}(\mathtt{ip}, \mathrm{T}_\xi(data)).\mathrm{T}_V(\xi, \mathrm{p}) \text{ where } \mathtt{ip} \notin \mathrm{VARS}(\mathrm{T}_\xi(data)) \cup \mathrm{VARS}(\mathrm{T}_V(\xi, \mathrm{p})) \quad \text{T5}$$

Consider the following derivation for expressions of the form $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{deliver}(data).\mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{del}(\hat{\mathtt{i}}, \mathrm{T}_\xi(data)) \xrightarrow{[\![\mathbf{del}(\hat{\mathtt{i}}, \mathrm{T}_\xi(data))]\!]} \checkmark}\,\text{Axiom}}{\mathbf{del}(\hat{\mathtt{i}}, \mathrm{T}_\xi(data)) \xrightarrow{\mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{del}(\hat{\mathtt{i}}, \mathrm{T}_\xi(data)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{\mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\,\text{Seq 1}}{\mathbf{del}(\mathtt{ip}, \mathrm{T}_\xi(data)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q})\,[\mathtt{ip} := \hat{\mathtt{i}}] \xrightarrow{\mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\,\text{Substitution}}{\textstyle\sum_{\mathtt{ip}} \mathbf{del}(\mathtt{ip}, \mathrm{T}_\xi(data)).\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{\mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\,\text{Sum 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{deliver}(ms).\mathrm{p}) \xrightarrow{\mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})}\,\text{T5}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{deliver}(ms).\mathrm{p}) \xrightarrow{\mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})}\,\text{Define } \sigma := \emptyset$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that $a \in \{\, \mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!]) \mid \hat{\mathtt{i}} : \mathrm{IP} \,\}$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$. There is one pair $(A_1, A_2) \in \mathcal{A}^\smile \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{deliver}(data).\mathrm{p}) \xrightarrow{A_1} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$:

$$(\{\, \mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!]) \mid \hat{\mathtt{i}} : \mathrm{IP} \,\}, \{\, \mathbf{deliver}(\xi(data)) \,\})$$

This pair satisfies the condition that $\xi, \mathbf{deliver}(data).\mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule DELIVER (T1)), which is sufficient to prove this particular base case.

Translation rule T6: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p}) = \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathrm{T}(\mathtt{msg}):\mathrm{MSG}} \mathbf{receive}(\mathtt{D},\mathtt{D'},\mathrm{T}(\mathtt{msg})).\mathrm{T}_{V \cup \{\mathtt{msg}\}}(\xi, \mathrm{p}) \text{ where } \mathtt{D},\mathtt{D'} \notin \mathrm{VARS}(\mathrm{T}_{V \cup \{\mathtt{msg}\}}(\xi, \mathrm{p})) \quad \text{T6}$$

Consider the following derivation for expressions of the form $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}',t_{\mathrm{U}(m)}) \xrightarrow{[\![\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}',t_{\mathrm{U}(m)})]\!]} \checkmark}\,\text{Axiom}}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}',t_{\mathrm{U}(m)}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],[\![t_{\mathrm{U}(m)}]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}',t_{\mathrm{U}(m)}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m))} \checkmark}\,\text{Lemma 4.4}}{\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}',t_{\mathrm{U}(m)}).\mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m))} \mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Seq 1}}{(\mathbf{receive}(\hat{\mathtt{D}},\hat{\mathtt{D}}',\mathrm{T}(\mathtt{msg})).\mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi,\mathrm{p}))[\mathrm{T}(\mathtt{msg}):=t_{\mathrm{U}(m)}] \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m))} \mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Lemma 4.3}}{(\mathbf{receive}(\mathtt{D},\mathtt{D'},\mathrm{T}(\mathtt{msg})).\mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi,\mathrm{p}))[\mathtt{D}:=\hat{\mathtt{D}},\mathtt{D'}:=\hat{\mathtt{D}}',\mathrm{T}(\mathtt{msg}):=t_{\mathrm{U}(m)}] \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m))} \mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Substitution}}{\textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathrm{T}(\mathtt{msg}):\mathrm{MSG}} \mathbf{receive}(\mathtt{D},\mathtt{D'},\mathrm{T}(\mathtt{msg})).\mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m))} \mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{Sum 2 (3 times)}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m))} \mathrm{T}_{W\cup\{\mathtt{msg}\}}(\xi[\mathtt{msg}:=m],\mathrm{p})}\,\text{T6}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,\mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{\mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m))} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})}\,\text{Define } \sigma = [\mathtt{msg}:=m]$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that $a = \tau$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})$. There is only one candidate for $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$, namely

$$(\{\, \mathbf{receive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}']\!],\mathrm{U}(m)) \mid \hat{\mathtt{D}},\hat{\mathtt{D}}' : \mathrm{Set(IP)} \,\},\{\, \mathbf{receive}(m) \,\})$$

This candidate satisfies the condition that $\xi, \mathbf{receive}(\mathtt{msg}).\mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule RECEIVE (T1)), which is sufficient to prove this particular base case.

Translation rule T7: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, [\![\mathtt{var}:=exp]\!]\ \mathrm{p}) = \textstyle\sum_{\mathtt{tmp}:\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathtt{tmp} = \mathrm{T}_\xi(exp)) \rightarrow \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var}) = \mathtt{tmp}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{V\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \quad \text{T7}$$
$$\text{where } \mathrm{T}(\mathtt{tmp}) \notin \mathrm{VARS}(\mathrm{T}_\xi(exp)) \cup \mathrm{VARS}(\mathrm{T}_{V\cup\{\mathtt{var}\}}(\xi,\mathrm{p}))$$

Consider the following derivation for expressions of the form $\mathrm{T}_V(\xi, [\![\mathtt{var}:=exp]\!]\ \mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}) \xrightarrow{[\![\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy})]\!]} \checkmark}\,\text{Axiom}}{\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \checkmark}\,\text{Definition 4.10}}{\mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Seq 1}}{[\![t_{\mathrm{U}(\xi(exp))} = t_{\mathrm{U}(\xi(exp))}]\!] = \mathit{true}}\,\text{Guard 2}}{(t_{\mathrm{U}(\xi(exp))} = t_{\mathrm{U}(\xi(exp))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Lemma 4.3}}{(\mathrm{T}(\mathtt{var}) = t_{\mathrm{U}(\xi(exp))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p})[\mathrm{T}(\mathtt{var}):=t_{\mathrm{U}(\xi(exp))}] \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Sum 2}}{\big((t_{\mathrm{U}(\xi(exp))} = t_{\mathrm{U}(\xi(exp))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p})\big)[\mathrm{T}(\mathtt{var}):=t_{\mathrm{U}(\xi(exp))}] \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Sum 2}}{\textstyle\sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var}) = t_{\mathrm{U}(\xi(exp))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Guard 2}}{[\![t_{\mathrm{U}(\xi(exp))} = t_{\mathrm{U}(\xi(exp))}]\!] = \mathit{true}}\,\text{Lemma 4.1}}{(t_{\mathrm{U}(\xi(exp))} = t_{\mathrm{U}(\xi(exp))}) \rightarrow \textstyle\sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var}) = t_{\mathrm{U}(\xi(exp))}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Lemma 4.1}}{(\mathtt{tmp} = \mathrm{T}_\xi(exp)) \rightarrow \textstyle\sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var}) = \mathtt{tmp}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Sum 2}}{\big((\mathtt{tmp} = \mathrm{T}_\xi(exp)) \rightarrow \textstyle\sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var}) = \mathtt{tmp}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p})\big)[\mathtt{tmp}:=t_{\mathrm{U}(\xi(exp))}] \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Sum 2}}{\textstyle\sum_{\mathtt{tmp}:\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathtt{tmp} = \mathrm{T}_\xi(exp)) \rightarrow \sum_{\mathrm{T}(\mathtt{var}):\mathrm{sort}(\mathrm{T}(\mathtt{var}))}(\mathrm{T}(\mathtt{var}) = \mathtt{tmp}) \rightarrow \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\,dummy}).\mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi,\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{T7}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,[\![\mathtt{var}:=exp]\!]\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi)\cup\{\mathtt{var}\}}(\xi[\mathtt{var}:=\xi(exp)],\mathrm{p})}\,\text{Define } \sigma = [\mathtt{var}:=\xi(exp)]$$
$$\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi,[\![\mathtt{var}:=exp]\!]\mathrm{p}) \xrightarrow{\mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])} \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that $a = \mathbf{t}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathtt{msg}_{\,dummy}]\!])$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma],\mathrm{p})$. There is only one candidate for $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in$

$A_1 \wedge \mathrm{T}_{\textsc{dom}(\xi)}(\xi, \llbracket \mathtt{var} := exp \rrbracket\ \mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$, namely

$$(\{\ \mathbf{t}(\textsc{u}(D), \textsc{u}(R), \textsc{u}(m))\ \}, \{\ \tau\ \})$$

This candidate satisfies the condition that $\xi, \llbracket \mathtt{var} := exp \rrbracket\ \mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule Assignment (T1)), which is sufficient to prove this particular base case.

Translation rule T10: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, [\phi]\mathrm{p}) = \textstyle\sum_{\textsc{t}(\mathrm{Fv}(\phi)\ \setminus\ V)} \textsc{t}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{V\ \cup\ \mathrm{Fv}(\phi)}(\xi, \mathrm{p}) \quad \mathrm{T10}$$

First, define σ such that $\textsc{dom}(\sigma) = \mathrm{Fv}(\phi) \setminus \textsc{dom}(\xi)$.
Now consider the following derivation for expressions of the form $\mathrm{T}_{\textsc{dom}(\xi)}(\xi, [\phi]\mathrm{p}) \xrightarrow{a} \mathrm{Q}$:

Axiom

$$\mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}) \xrightarrow{\llbracket \mathbf{t}(\emptyset,\emptyset,\mathtt{msg}_{\ dummy}) \rrbracket} \checkmark$$

Definition 4.10

$$\mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \checkmark$$

Seq 1

$$\mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\zeta)}(\zeta, \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\zeta)}(\zeta, \mathrm{p})$$

$\llbracket \textsc{t}_\zeta(\phi) \rrbracket = true$ — Guard 2

$$\textsc{t}_\zeta(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\zeta)}(\zeta, \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\zeta)}(\zeta, \mathrm{p})$$

Definition of ζ (5 times)

$$\textsc{t}_{\xi[\sigma]}(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

Definition of σ

$$\textsc{t}_{\xi[\mathtt{q}_1 := e_1, \cdots, \mathtt{q}_n := e_n]}(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

Lemma 4.2

$$\textsc{t}_\xi(\phi)[\textsc{t}(\mathtt{q}_1) := t_{\textsc{u}(e_1)}, \cdots, \textsc{t}(\mathtt{q}_n) := t_{\textsc{u}(e_n)}] \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

Definition of σ

$$\textsc{t}_\xi(\phi)[\textsc{t}(\mathtt{q}_1) := t_{\textsc{u}(e_1)}, \cdots, \textsc{t}(\mathtt{q}_n) := t_{\textsc{u}(e_n)}] \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\mathtt{q}_1 := e_1, \cdots, \mathtt{q}_n := e_n], \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

Lemma 4.3

$$\textsc{t}_\xi(\phi)[\textsc{t}(\mathtt{q}_1) := t_{\textsc{u}(e_1)}, \cdots, \textsc{t}(\mathtt{q}_n) := t_{\textsc{u}(e_n)}] \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi, \mathrm{p})[\textsc{t}(\mathtt{q}_1) := t_{\textsc{u}(e_1)}, \cdots, \textsc{t}(\mathtt{q}_n) := t_{\textsc{u}(e_n)}] \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

$$(\textsc{t}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi, \mathrm{p}))[\textsc{t}(\mathtt{q}_1) := t_{\textsc{u}(e_1)}, \cdots, \textsc{t}(\mathtt{q}_n) := t_{\textsc{u}(e_n)}] \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

Sum 2

$$\textstyle\sum_{\textsc{t}(\textsc{dom}(\sigma))} \textsc{t}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi, \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

Definition of σ (2 times)

$$\textstyle\sum_{\textsc{t}(\mathrm{Fv}(\phi)\ \setminus\ \textsc{dom}(\xi))} \textsc{t}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi) \cup (\mathrm{Fv}(\phi)\ \setminus\ \textsc{dom}(\xi))}(\xi, \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

$$\textstyle\sum_{\textsc{t}(\mathrm{Fv}(\phi)\ \setminus\ \textsc{dom}(\xi))} \textsc{t}_\xi(\phi) \rightarrow \mathbf{t}(\emptyset, \emptyset, \mathtt{msg}_{\ dummy}).\mathrm{T}_{\textsc{dom}(\xi) \cup \mathrm{Fv}(\phi)}(\xi, \mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

T10

$$\mathrm{T}_{\textsc{dom}(\xi)}(\xi, [\phi]\mathrm{p}) \xrightarrow{\mathbf{t}(\llbracket\emptyset\rrbracket,\llbracket\emptyset\rrbracket,\llbracket\mathtt{msg}_{\ dummy}\rrbracket)} \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$$

This derivation is a representative derivation without alternatives (see Definition 4.9). It follows that $a = \mathbf{t}(\llbracket\emptyset\rrbracket, \llbracket\emptyset\rrbracket, \llbracket\mathtt{msg}_{\ dummy}\rrbracket)$ and $\mathrm{Q} = \mathrm{T}_{\textsc{dom}(\xi[\sigma])}(\xi[\sigma], \mathrm{p})$. There is only one candidate for $(A_1, A_2) \in \mathcal{A}^{\smile}$. $a \in A_1 \wedge \mathrm{T}_{\textsc{dom}(\xi)}(\xi, [\phi]\mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$, namely

$$\left(\left\{\ \mathbf{t}(\llbracket\emptyset\rrbracket, \llbracket\emptyset\rrbracket, \llbracket\mathtt{msg}_{\ dummy}\rrbracket)\ \right\}, \{\ \tau\ \}\right)$$

This candidate satisfies the condition that $\xi, [\phi]\mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}$ can be derived for all $a' \in A_2$ (via AWN inference rule Guard (T1)), which is sufficient to prove this particular base case.

2 Induction steps

Translation rule T8: The translation function T_V is partially defined by translation rule

$$\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) = \mathrm{X}(\textsc{t}_\xi(exp_1), \cdots, \textsc{t}_\xi(exp_n)) \quad \mathrm{T8}$$

$$\text{where } \mathrm{T}(\mathrm{X}(\mathtt{var}_1, \cdots, \mathtt{var}_n) \stackrel{\text{def}}{=} \mathrm{p}) = \mathrm{X}(\textsc{t}(\mathtt{var}_1) : \mathrm{sort}(\textsc{t}_\xi(exp_1)), \cdots, \textsc{t}(\mathtt{var}_n) : \mathrm{sort}(\textsc{t}_\xi(exp_n))) \stackrel{\text{def}}{=} \mathrm{T}_{\{\mathtt{var}_1, \cdots, \mathtt{var}_n\}}(\emptyset, \mathrm{p})$$

The induction hypothesis states that recursions T_V occurring in $\mathrm{X}(\mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}_\xi(exp_n))$ are related. In particular, the induction hypothesis is used here to relate arbitrary instantiations of process calls:

$$\begin{aligned}\forall a, \mathrm{Q} \,.\, &\mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p}) \xrightarrow{a} \mathrm{Q} \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\smile}, \mathrm{q}, \sigma \,.\, a \in A_1 \\ &\wedge \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p}) \xrightarrow{A_1} \mathrm{Q} \\ &\wedge \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p} \xrightarrow{A_2} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q} \\ &\wedge \mathrm{Q} = \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}\cup \mathrm{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})\end{aligned}$$

The following derivation in AWN can be based on the third line of the instantiated induction hypothesis:

$$\dfrac{\dfrac{}{\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p} \xrightarrow{a'} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q}}\ \text{Induction hypothesis} \quad \text{Definition of X}}{\xi, \mathrm{X}(exp_1, \cdots, exp_n) \xrightarrow{a'} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q}}\ \textsc{Recursion (T1)}$$

This derivation holds for all $a' \in A_2$. Similarly, in mCRL2, it happens to be the case that

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p}) \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}\cup \mathrm{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\ \text{Induction hypothesis}}{\mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}}(\emptyset, \mathrm{p})[\mathrm{r}(\mathtt{d}_1) := t_{\mathrm{U}(\xi(exp_1))}, \cdots, \mathrm{r}(\mathtt{d}_n) := t_{\mathrm{U}(\xi(exp_n))}] \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}\cup \mathrm{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\ \text{Lemma 4.3}}{\mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}}(\emptyset, \mathrm{p})[\mathrm{r}(\mathtt{d}_1) := \mathrm{T}_\xi(exp_1), \cdots, \mathrm{r}(\mathtt{d}_n) := \mathrm{T}_\xi(exp_n)] \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}\cup \mathrm{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\ \text{Lemma 4.1}}{\mathrm{X}(\mathrm{T}_\xi(exp_1), \cdots, \mathrm{T}_\xi(exp_n)) \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}\cup \mathrm{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\ \textsc{Recursion 2}}{\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{a} \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}\cup \mathrm{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})}\ \text{T8}\quad \text{Definition of X}$$

for all $a \in A_1$. This derivation is a representative derivation without alternatives (see Definition 4.9). Because the induction hypothesis is used to express the relationship between *arbitrary* instantiations of process calls, the first line of this derivation plus the inference rules of mCRL2 must be sufficient to generate all possible behavior of an mCRL2 process call. The actions available to $\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n))$ in mCRL2 can therefore clearly be mimicked by AWN, and so both derivations can be combined into an new equation matching the induction hypothesis:

$$\begin{aligned}\forall a, \mathrm{Q} \,.\, &\mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{a} \mathrm{Q} \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\smile}, \mathrm{q}, \sigma \,.\, a \in A_1 \\ &\wedge \mathrm{T}_V(\xi, \mathrm{X}(exp_1, \cdots, exp_n)) \xrightarrow{A_1} \mathrm{Q} \\ &\wedge \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)], \mathrm{p} \xrightarrow{A_2} \emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q} \\ &\wedge \mathrm{Q} = \mathrm{T}_{\{\mathtt{d}_1,\cdots,\mathtt{d}_n\}\cup \mathrm{DOM}(\sigma)}(\emptyset[\mathtt{d}_1 := \xi(exp_1), \cdots, \mathtt{d}_n := \xi(exp_n)][\sigma], \mathrm{q})\end{aligned}$$

This confirms the induction step of the proof of Lemma 4.7.

Translation rule T9: The translation function $\mathrm{T}_{\mathrm{DOM}(\xi)}$ is partially defined by translation rule

$$\mathrm{T}_V(\xi, \mathrm{p} + \mathrm{q}) = \mathrm{T}_V(\xi, \mathrm{p}) + \mathrm{T}_V(\xi, \mathrm{q}) \quad \text{T9}$$

Suppose that $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}+\mathrm{q}) \xrightarrow{a} \mathrm{P}'$ for some a and P'. According to the semantics of mCRL2 (in particular inference rules Choice 2 and Choice 4) this can only be the case if $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{P}'$ or $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{a} \mathrm{P}'$ or both:

$$\dfrac{\dfrac{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{P}'}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) + \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{a} \mathrm{P}'}\ \textsc{Choice 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}+\mathrm{q}) \xrightarrow{a} \mathrm{P}'}\ \text{T9} \qquad \dfrac{\dfrac{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{a} \mathrm{P}'}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) + \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{a} \mathrm{P}'}\ \textsc{Choice 4}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}+\mathrm{q}) \xrightarrow{a} \mathrm{P}'}\ \text{T9}$$

That is,

$$\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}+\mathrm{q}) \xrightarrow{a} \mathrm{P}' \Rightarrow \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{P}' \vee \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{a} \mathrm{P}' \tag{B.1}$$

According to the induction hypothesis, $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{P}'$ implies that there must exist some $(A_1, A_2) \in \mathcal{A}^{\smile}$ and p' such that $a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{p} \xrightarrow{A_2} \mathrm{p}' \wedge \mathrm{P}' = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}')$, which makes exactly the following derivations possible (in mCRL2 and AWN, respectively):

$$\dfrac{\dfrac{\dfrac{}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{P}'}\ \text{Induction hypothesis}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) + \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{A_1} \mathrm{P}'}\ \textsc{Choice 2}}{\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p} + \mathrm{q}) \xrightarrow{A_1} \mathrm{P}'}\ \mathrm{T9} \qquad \text{and} \qquad \dfrac{\dfrac{}{\mathrm{p} \xrightarrow{A_2} \mathrm{p}'}\ \text{Induction hypothesis}}{\mathrm{p} + \mathrm{q} \xrightarrow{A_2} \mathrm{p}'}\ \textsc{Choice (T1-1)}$$

and therefore

$$\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{P}' \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\vee}, \mathrm{p}' \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p} + \mathrm{q}) \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{p} + \mathrm{q} \xrightarrow{A_2} \mathrm{p}' \wedge \mathrm{P}' = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}')$$

Similar reasoning can be applied to $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{P}'$:

$$\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{q}) \xrightarrow{a} \mathrm{P}' \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\vee}, \mathrm{p}' \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p} + \mathrm{q}) \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{p} + \mathrm{q} \xrightarrow{A_2} \mathrm{p}' \wedge \mathrm{P}' = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}')$$

Combining both of those results via Equation B.1, it then becomes clear that

$$\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p} + \mathrm{q}) \xrightarrow{a} \mathrm{P}' \Rightarrow \exists (A_1, A_2) \in \mathcal{A}^{\vee}, \mathrm{p}' \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p} + \mathrm{q}) \xrightarrow{A_1} \mathrm{P}' \wedge \mathrm{p} + \mathrm{q} \xrightarrow{A_2} \mathrm{p}' \wedge \mathrm{P}' = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}')$$

which is sufficient to prove this particular base case.

Appendix C

Complete proof of Lemma 4.8

1 Base cases

Translation rule T11: This translation rule is an exceptional case because a process definition by itself cannot do any transitions, and providing a proof for Lemma 4.8 is therefore nonsensical. Related to T11 is process recursion, for which a proof *does* make sense; see the Lemma 4.7-proof for Translation rule T8.

Translation rule T12: The translation function T is partially defined by translation rule T12:

$$\mathrm{T}(\xi, \mathrm{p}) = \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p})$$

Suppose that $\mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{a} \mathrm{Q}$. Then, according to Lemma 4.7, there exists some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}_{\mathrm{DOM}(\xi)}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$ such that $\xi, \mathrm{p} \xrightarrow{a'} \xi[\sigma], \mathrm{p}'$ for all $a' \in A_2$ and $\mathrm{Q} = \mathrm{T}_{\mathrm{DOM}(\xi[\sigma])}(\xi[\sigma], \mathrm{p}')$.
Define $\zeta = \xi[\sigma]$. Q can then be rewritten to $\mathrm{T}_{\mathrm{DOM}(\zeta)}(\zeta, \mathrm{p}')$ which, as stated by rule T12, equals $\mathrm{T}(\zeta, \mathrm{p}')$. As a result, there must exist some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}(\xi, \mathrm{p}) \xrightarrow{A_1} \mathrm{Q}$ such that $\xi, \mathrm{p} \xrightarrow{a'} \zeta, \mathrm{p}'$ for all $a' \in A_2$ and $\mathrm{Q} = \mathrm{T}(\zeta, \mathrm{p}')$, proving this particular base case.

2 Induction steps

Translation rule T13: The translation function T is partially defined by translation rule

$$\mathrm{T}(\mathrm{P} \lll \mathrm{Q}) = \nabla_V \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{t}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \text{ where } V = \{\mathbf{t}, \mathbf{cast}, \neg\mathbf{uni}, \mathbf{send}, \mathbf{del}, \mathbf{receive}\} \quad \text{T13}$$

Consider Pairs 2.10 and 2.8 from Table 2.6:

$$(\{\ \mathbf{receive}(m)\ \}, \{\ \mathbf{receive}([\![\hat{\mathrm{D}}]\!], [\![\hat{\mathrm{D}}\text{'}]\!], \mathrm{U}(m)) \mid \hat{\mathrm{D}}, \hat{\mathrm{D}}\text{'} : \mathrm{Set(IP)}\ \})$$
$$(\{\ \mathbf{send}(\xi(ms))\ \}, \{\ \mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \})$$

Let R_1 and S_1 be defined as the second members of these pairs; that is, let

$$R_1 \stackrel{\text{def}}{=} \{\ \mathbf{receive}([\![\hat{\mathrm{D}}]\!], [\![\hat{\mathrm{D}}\text{'}]\!], \mathrm{U}(m)) \mid \hat{\mathrm{D}}, \hat{\mathrm{D}}\text{'} : \mathrm{Set(IP)}\ \}$$
$$S_1 \stackrel{\text{def}}{=} \{\ \mathbf{send}([\![\emptyset]\!], [\![\emptyset]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \}$$

for some m, $\xi(ms)$: MSG.
The following cases are distinguished:

1: T(P) does a transition $a \in R_1$, T(Q) does a transition $b \in S_1$, and $\mathrm{U}(m) = [\![\mathrm{T}_\xi(ms)]\!]$.

2: $\mathrm{T}(\mathrm{P})$ does a transition $a \in R_1$, $\mathrm{T}(\mathrm{Q})$ does a transition $b \in S_1$, and $\mathrm{U}(m) \neq [\![\mathrm{T}_\xi(ms)]\!]$.
3: $\mathrm{T}(\mathrm{P})$ does a transition a, $\mathrm{T}(\mathrm{Q})$ does a transition b, and $\underline{a} \neq$ **receive** or $\underline{b} \neq$ **send** or both.
4: $\mathrm{T}(\mathrm{P})$ does a transition a where $\underline{a} =$ **receive** and $\mathrm{T}(\mathrm{Q})$ does not do a transition.
5: $\mathrm{T}(\mathrm{P})$ does a transition a where $\underline{a} \neq$ **receive** and $\mathrm{T}(\mathrm{Q})$ does not do a transition.
6: $\mathrm{T}(\mathrm{P})$ does not do a transition and $\mathrm{T}(\mathrm{Q})$ does a transition b where $\underline{b} =$ **send**.
7: $\mathrm{T}(\mathrm{P})$ does not do a transition and $\mathrm{T}(\mathrm{Q})$ does a transition b where $\underline{b} \neq$ **send**.

Note that these cases cover all combinations of behavior of $\mathrm{T}(\mathrm{P})$ and $\mathrm{T}(\mathrm{Q})$.
The proof is provided below for each of the cases:

1: Suppose that $\mathrm{T}(\mathrm{P})$ does a transition $a \in R_1$, $\mathrm{T}(\mathrm{Q})$ does a transition $b \in S_1$, and $\mathrm{U}(m) = [\![\mathrm{T}_\xi(ms)]\!]$. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from R_1 or S_1, it can first be concluded that

$$\mathrm{T}(\mathrm{P}) \xrightarrow{R_1} \mathrm{T}(\mathrm{P}') \wedge \mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}' \wedge \mathrm{T}(\mathrm{Q}) \xrightarrow{S_1} \mathrm{T}(\mathrm{Q}') \wedge \mathrm{Q} \xrightarrow{\mathbf{send}(\xi(ms))} \mathrm{Q}'$$

Because $\mathrm{U}(m) = [\![\mathrm{T}_\xi(ms)]\!] \overset{\text{Lemma 4.5}}{\Longleftrightarrow} m = \xi(ms)$ this becomes

$$\mathrm{T}(\mathrm{P}) \xrightarrow{R_1} \mathrm{T}(\mathrm{P}') \wedge \mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}' \wedge \mathrm{T}(\mathrm{Q}) \xrightarrow{S_1} \mathrm{T}(\mathrm{Q}') \wedge \mathrm{Q} \xrightarrow{\mathbf{send}(m)} \mathrm{Q}'$$

The mCRL2 derivation in the Lemma 4.6-proof for Parallel (T2-3) shows how the first and third conjunct are sufficient to prove that $\mathrm{T}(\mathrm{P} \between \mathrm{Q}) \xrightarrow{a} \mathrm{T}(\mathrm{P}' \between \mathrm{Q}')$ for all $a \in \{\ \mathbf{t}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))\ \}$ for some D, R where $R \subseteq D$.
On the AWN side, the second and fourth conjunct can be used as premises in

$$\forall m \in \mathrm{MSG}\ \frac{\mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}' \qquad \mathrm{Q} \xrightarrow{\mathbf{send}(m)} \mathrm{Q}'}{\mathrm{P} \between \mathrm{Q} \xrightarrow{\tau} \mathrm{P}' \between \mathrm{Q}'}\ \textsc{Parallel (T2-3)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^\smile$ that satisfies $\mathbf{t}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m)) \in A_1 \wedge \mathrm{T}(\mathrm{P} \between \mathrm{Q}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}' \between \mathrm{Q}') \wedge \mathrm{P} \between \mathrm{Q} \xrightarrow{A_2} \mathrm{P}' \between \mathrm{Q}'$, and the converse of Pair 2.3 satisfies this requirement.

2: Suppose that $\mathrm{T}(\mathrm{P})$ does a transition $a \in R_1$, $\mathrm{T}(\mathrm{Q})$ does a transition $b \in S_1$, and $\mathrm{U}(m) \neq [\![\mathrm{T}_\xi(ms)]\!]$. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from R_1 or S_1, it can first be concluded that

$$\mathrm{T}(\mathrm{P}) \xrightarrow{R_1} \mathrm{T}(\mathrm{P}') \wedge \mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}' \wedge \mathrm{T}(\mathrm{Q}) \xrightarrow{S_1} \mathrm{T}(\mathrm{Q}') \wedge \mathrm{Q} \xrightarrow{\mathbf{send}(\xi(ms))} \mathrm{Q}'$$

where $\mathrm{U}(m) \neq [\![\mathrm{T}_\xi(ms)]\!] \overset{\text{Lemma 4.5}}{\Longleftrightarrow} m \neq \xi(ms)$.
The mCRL2 derivation below shows where the attempt to generate behavior for $\mathrm{T}(\mathrm{P} \between \mathrm{Q}) \xrightarrow{\tau} \mathrm{T}(\mathrm{P}' \between \mathrm{Q}')$ fails:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{receive}([\![\hat{\mathrm{D}}]\!],[\![\hat{\mathrm{D}}']\!],\mathrm{U}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Induction hypothesis}}{\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{receive}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))} \mathrm{T}(\mathrm{P}')}\ \text{Choose } \hat{\mathrm{D}} = \hat{\mathrm{D}}' = \emptyset}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\{\mathbf{receive}\to\mathbf{r}\}\bullet\mathbf{receive}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\ \textsc{Rename 2}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\ \text{Apply } \{\mathbf{receive}\to\mathbf{r}\}\bullet$$

$$\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{send}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \mathrm{T}(\mathrm{Q}')}\ \text{Induction hypothesis}}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\{\mathbf{send}\to\mathbf{s}\}\bullet\mathbf{send}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \textsc{Rename 2}}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \text{Apply } \{\mathbf{send}\to\mathbf{s}\}\bullet$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\ \textit{(see above)} \quad \dfrac{}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \textit{(see above)}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))|\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \textsc{Par 3}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))|\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!]))} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \textsc{Comm 2}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))|\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \text{Apply } \gamma$$

$$\dfrac{\dfrac{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\{\mathbf{rs}\to\mathbf{t}\}\bullet\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))|\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \textsc{Rename 2}}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\mathbf{r}([\![\emptyset]\!],[\![\emptyset]\!],\mathrm{U}(m))|\mathbf{s}([\![\emptyset]\!],[\![\emptyset]\!],[\![\mathrm{T}_\xi(ms)]\!])} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}'))}\ \text{Apply } \{\mathbf{rs}\to\mathbf{t}\}\bullet$$

The ALLOW 2 operator cannot be applied next (like in the previous case) because $\mathbf{r}|\mathbf{s} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}(\mathrm{M} \lll \mathrm{N})$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

3: Suppose that $\mathrm{T}(\mathrm{P})$ does a transition a, $\mathrm{T}(\mathrm{Q})$ does a transition b, and $\underline{a} \neq$ **receive** or $\underline{b} \neq$ **send** or both.
This situation fails to generate behavior for $\mathrm{T}(\mathrm{P} \lll \mathrm{Q})$ because no derivation similar to the one in the Lemma 4.6-proof for Parallel (T2-3) is possible:

a. The a action may be able to be converted to an **r** action and the b action may be able to be converted to an **s** action, but never both at the same time;
b. The $a|b$ action produced by $\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})$ is (without arguments) never equal to $\mathbf{r}|\mathbf{s}$ (that is, $\underline{a|b} \neq \mathbf{r}|\mathbf{s}$);
c. Because there is never an $\mathbf{r}|\mathbf{s}$ action, the communication operator $\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}$ does not generate an **rs** action but leaves $a|b$ intact;
d. The renaming operator $\rho_{\{\mathbf{rs}\to\mathbf{t}\}}$ has no effect because $\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))$ does not produce an **rs** action;
e. Among others, the allow operator ∇_V blocks all multi-actions, and so $a|b$ is blocked.

Since $\mathrm{T}(\mathrm{P}) \lll \mathrm{T}(\mathrm{Q})$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

4: Suppose that $\mathrm{T}(\mathrm{P})$ does a transition a with $\underline{a} =$ **receive** and $\mathrm{T}(\mathrm{Q})$ does not do a transition.
This situation fails to generate behavior for $\mathrm{T}(\mathrm{P} \lll \mathrm{Q})$ because no derivation similar to the one in the Lemma 4.6-proof for Parallel (T2-3) is possible:

a. The a action of $\mathrm{T}(\mathrm{P})$ is renamed to an **r** action by the renaming operator $\rho_{\{\mathbf{receive}\to\mathbf{r}\}}$;
b. Because $\mathbf{r} \neq \mathbf{r}|\mathbf{s}$, the communication operator $\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}$ does not generate an **rs** action but leaves the **r** action intact;
c. The renaming operator $\rho_{\{\mathbf{rs}\to\mathbf{t}\}}$ has no effect because $\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))$ does not produce an **rs** action;
d. The allow operator ∇_V cannot be applied because $\mathbf{r} \notin V \cup \{\tau\}$.

Since $\mathrm{T}(\mathrm{P}) \lll \mathrm{T}(\mathrm{Q})$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

5: Suppose that $\mathrm{T}(\mathrm{P})$ does a transition a with $\underline{a} \neq$ **receive** and $\mathrm{T}(\mathrm{Q})$ does not do a transition. The induction hypothesis states that

$$\exists (A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}') \wedge \mathrm{P} \xrightarrow{A_2} \mathrm{P}'$$

Define $\mathcal{A}'$ as a subset of $\mathcal{A}^{\smile}$ that contains the possible values of $(A_1, A_2) \in \mathcal{A}^{\smile}$ where A_1 are actions that $\mathrm{T}(\mathrm{P} \lll \mathrm{Q})$ can perform in mCRL2 and where A_2 are the actions that AWN should use to mimic the actions in A_1 in order for this case to be proven:

$$\mathcal{A}' \stackrel{\text{def}}{=} \left\{ (A_1, A_2) \;\middle|\; (A_1, A_2) \in \mathcal{A}^{\smile}, \quad \mathrm{T}(\mathrm{P} \lll \mathrm{Q}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}' \lll \mathrm{Q}) \right\}$$

The following derivation is possible in mCRL2 for all $a \in A_1$ such that $(A_1, A_2) \in \mathcal{A}'$:

$$\underline{a} \in V \cup \{\tau\} \supseteq W \;\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{P}) \xrightarrow{a} \mathrm{T}(\mathrm{P}')}\text{ Induction hypothesis}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{\{\mathbf{receive}\to\mathbf{r}\}\bullet a} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\text{ RENAME 2}}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \xrightarrow{a} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}')}\text{ Apply } \{\mathbf{receive}\to\mathbf{r}\}\bullet}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{a} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})}\text{ PAR 2}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(a)} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ COMM 2}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ Apply } \gamma}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\theta_{\{\mathbf{rs}\}}(a)} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ RENAME 2}}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ Apply } \{\mathbf{rs}\to\mathbf{t}\}\bullet}{\nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{a} \nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \mid\mid \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\text{ ALLOW 2}}{\mathrm{T}(\mathrm{P} \lll \mathrm{Q}) \xrightarrow{a} \mathrm{T}(\mathrm{P}' \lll \mathrm{Q})}\text{ T13}$$

The derivation above is valid only for $a \in A_1$ such that $\underline{a} \subseteq V \cup \{\tau\}$. Under the constraints of this case, the derivation is also a representative derivation without alternatives (see Definition 4.9). Consequently, $\mathcal{A}'$ is as follows:

$$
\begin{aligned}
\{\ & \\
& (\{\ \mathbf{t}(\upsilon(D), \upsilon(R), \upsilon(m))\ \}, \{\ \tau\ \}), \\
& (\{\ \mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)\ \mid \hat{\mathtt{D}} : \mathrm{Set(IP)}\ \}, \{\ \mathbf{broadcast}(\xi(ms))\ \}), \\
& (\{\ \mathbf{cast}(\llbracket \mathrm{T}_\xi(dests) \rrbracket, \llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)\ \mid \hat{\mathtt{D}} : \mathrm{Set(IP)}\ \}, \{\ \mathbf{groupcast}(\xi(dests), \xi(ms))\ \}), \\
& (\{\ \mathbf{cast}(\llbracket \mathrm{T}_\xi(\{dest\}) \rrbracket, \llbracket \mathrm{T}_\xi(\{dest\}) \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)\ \}, \{\ \mathbf{unicast}(\xi(dest), \xi(ms))\ \}), \\
& (\{\ \neg\mathbf{uni}(\llbracket \mathrm{T}_\xi(\{dest\}) \rrbracket, \llbracket \mathrm{T}_\xi(\{dest\}) \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)\ \}, \{\ \neg\mathbf{unicast}(\xi(dest), \xi(ms))\ \}), \\
& (\{\ \mathbf{send}(\llbracket \emptyset \rrbracket, \llbracket \emptyset \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)\ \}, \{\ \mathbf{send}(\xi(ms))\ \}), \\
& (\{\ \mathbf{del}(\llbracket \hat{\mathtt{i}} \rrbracket, \llbracket \mathrm{T}_\xi(data) \rrbracket)\ \mid \hat{\mathtt{i}} : \mathrm{IP}\ \}, \{\ \mathbf{deliver}(\xi(data))\ \}) \\
\mid\ & m, \xi(ms) : \mathrm{MSG};\quad dest : \mathrm{IP};\quad \xi(data) : \mathrm{DATA};\quad dests, R, D : \mathrm{Set(IP)};\quad R \subseteq D\quad \}
\end{aligned}
$$

For all $(A_1, A_2) \in \mathcal{A}'$ the actions A_1 in mCRL2 can be mimicked by the actions A_2 in AWN by means of the inference rule

$$\forall a' \neq \mathbf{receive}(m)\ \frac{\mathrm{P} \xrightarrow{a'} \mathrm{P}'}{\mathrm{P} \between \mathrm{Q} \xrightarrow{a'} \mathrm{P}' \between \mathrm{Q}}\ \textsc{Parallel (T2-1)}$$

which is valid for all $a' \in A_2$. Therefore the induction hypothesis holds for this case.

6: Suppose that $\mathrm{T}(\mathrm{P})$ does not do a transition and $\mathrm{T}(\mathrm{Q})$ does a transition b with $\underline{b} = \mathbf{send}$.
This situation fails to generate behavior for $\mathrm{T}(\mathrm{P} \between \mathrm{Q})$ because no derivation similar to the one in the Lemma 4.6-proof for Parallel (T2-3) is possible:

a. The a action of $\mathrm{T}(\mathrm{P})$ is renamed to an **s** action by the renaming operator $\rho_{\{\mathbf{send}\to\mathbf{s}\}}$;
b. Because $\mathbf{s} \neq \mathbf{r}|\mathbf{s}$, the communication operator $\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}$ does not generate an **rs** action but leaves the **s** action intact;
c. The renaming operator $\rho_{\{\mathbf{rs}\to\mathbf{t}\}}$ has no effect because $\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))$ does not produce an **rs** action;
d. The allow operator ∇_V cannot be applied because $\mathbf{s} \notin V \cup \{\tau\}$.

Since $\mathrm{T}(\mathrm{P}) \between \mathrm{T}(\mathrm{Q})$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

7: Suppose that $\mathrm{T}(\mathrm{P})$ does not do a transition and $\mathrm{T}(\mathrm{Q})$ does a transition b with $\underline{b} \neq \mathbf{send}$. The induction hypothesis states that

$$\exists (B_1, B_2) \in \mathcal{A}^{\smile}\ .\ b \in B_1 \wedge \mathrm{T}(\mathrm{Q}) \xrightarrow{B_1} \mathrm{T}(\mathrm{Q}') \wedge \mathrm{Q} \xrightarrow{B_2} \mathrm{Q}'$$

Define $\mathcal{A}'$ as a subset of $\mathcal{A}^{\smile}$ that contains the possible values of $(B_1, B_2) \in \mathcal{A}^{\smile}$ where B_1 are actions that $\mathrm{T}(\mathrm{P} \between \mathrm{Q})$ can perform in mCRL2 and where B_2 are the actions that AWN should use to mimic the actions in B_1 in order for this case to be proven:

$$\mathcal{A}' \stackrel{\text{def}}{=} \left\{ (B_1, B_2) \;\middle|\; (B_1, B_2) \in \mathcal{A}^{\smile},\quad \mathrm{T}(\mathrm{P} \between \mathrm{Q}) \xrightarrow{B_1} \mathrm{T}(\mathrm{P} \between \mathrm{Q}') \right\}$$

The following derivation is possible in mCRL2 for all $b \in B_1$ such that $(B_1, B_2) \in \mathcal{A}'$:

$$
\underline{b} \in V \cup \{\tau\} \supseteq W\ \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{Q}) \xrightarrow{b} \mathrm{T}(\mathrm{Q}')}\ \text{Induction hypothesis}}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{\{\mathbf{send}\to\mathbf{s}\}\bullet b} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \textsc{Rename 2}}{\rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{b} \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}')}\ \text{Apply } \{\mathbf{send}\to\mathbf{s}\}\bullet}{\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}) \xrightarrow{b} \rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})}\ \textsc{Par 5}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\gamma(\mathbf{r}|\mathbf{s}\to\mathbf{rs})(b)} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\ \textsc{Comm 2}}{\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{b} \Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\ \text{Apply } \gamma}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{\theta_{\{\mathbf{rs}\}}(b)} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\ \textsc{Rename 2}}{\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{b} \rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\ \text{Apply } \{\mathbf{rs}\to\mathbf{t}\}\bullet}{\nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}) \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q})) \xrightarrow{b} \nabla_V\rho_{\{\mathbf{rs}\to\mathbf{t}\}}\Gamma_{\{\mathbf{r}|\mathbf{s}\to\mathbf{rs}\}}(\rho_{\{\mathbf{receive}\to\mathbf{r}\}}\mathrm{T}(\mathrm{P}') \parallel \rho_{\{\mathbf{send}\to\mathbf{s}\}}\mathrm{T}(\mathrm{Q}))}\ \textsc{Allow 2}}{\mathrm{T}(\mathrm{P} \between \mathrm{Q}) \xrightarrow{b} \mathrm{T}(\mathrm{P}' \between \mathrm{Q})}\ \textsc{T13}
$$

The derivation above is valid only for $b \in B_1$ such that $\underline{b} \in V \cup \{\tau\}$. Under the constraints of this case, the derivation is also a representative derivation without alternatives (see Definition 4.9). Consequently, $\mathcal{A}'$ is as follows:

$$
\begin{aligned}
\{\ & \\
& (\{\ \mathbf{t}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m))\ \}, \{\ \tau\ \}), \\
& (\{\ \mathbf{cast}([\![\mathcal{U}_{\mathrm{IP}}]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])\ |\ \hat{\mathtt{D}} : \mathrm{Set(IP)}\ \}, \{\ \mathbf{broadcast}(\xi(ms))\ \}), \\
& (\{\ \mathbf{cast}([\![\mathrm{T}_\xi(dests)]\!], [\![\hat{\mathtt{D}}]\!], [\![\mathrm{T}_\xi(ms)]\!])\ |\ \hat{\mathtt{D}} : \mathrm{Set(IP)}\ \}, \{\ \mathbf{groupcast}(\xi(dests), \xi(ms))\ \}), \\
& (\{\ \mathbf{cast}([\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \}, \{\ \mathbf{unicast}(\xi(dest), \xi(ms))\ \}), \\
& (\{\ \neg\mathbf{uni}([\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(\{dest\})]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \}, \{\ \neg\mathbf{unicast}(\xi(dest), \xi(ms))\ \}), \\
& (\{\ \mathbf{receive}([\![\hat{\mathtt{D}}]\!], [\![\hat{\mathtt{D}}\mathtt{'}]\!], \mathrm{U}(m))\ |\ \hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'} : \mathrm{Set(IP)}\ \}, \{\ \mathbf{receive}(m)\ \}), \\
& (\{\ \mathbf{del}([\![\hat{\mathtt{i}}]\!], [\![\mathrm{T}_\xi(data)]\!])\ |\ \hat{\mathtt{i}} : \mathrm{IP}\ \}, \{\ \mathbf{deliver}(\xi(data))\ \}) \\
& \\
|\quad & m, \xi(ms) : \mathrm{MSG}; dest : \mathrm{IP};\quad \xi(data) : \mathrm{DATA};\quad dests, R, D : \mathrm{Set(IP)};\quad R \subseteq D\quad \}
\end{aligned}
$$

For all $(B_1, B_2) \in \mathcal{A}'$ the actions B_1 in mCRL2 can be mimicked by the actions B_2 in AWN by means of the inference rule

$$\forall b' \neq \mathbf{send}(m)\ \frac{\mathrm{Q} \xrightarrow{b'} \mathrm{Q}'}{\mathrm{P} \between \mathrm{Q} \xrightarrow{b'} \mathrm{P} \between \mathrm{Q}'}\ \textsc{Parallel (T2-2)}$$

which is valid for all $b' \in B_2$. Therefore the induction hypothesis holds for this case.

The derivations in mCRL2 from the cases listed above are representative derivations that collectively have no alternatives (see Definition 4.9): there are no other derivations with different or differently ordered one-way steps that yield different behavior of process expression $\mathrm{T}(\mathrm{P} \between \mathrm{Q})$. Therefore the induction hypothesis generally holds for this expression.

Translation rule T14: **(*This proof will be rewritten, and therefore τ has not yet been replaced by t.*)**
The translation function T is partially defined by translation rule

$$
\begin{aligned}
\mathrm{T}(ip : \mathrm{P} : R) &= \nabla_V \Gamma_C(\mathrm{T}(\mathrm{P}) \mid\mid \mathrm{G}(t_{\mathrm{U}(ip)}, t_{\mathrm{U}(R)})) \quad \text{T14} \\
&\text{where } V = \{\mathbf{t}, \mathbf{starcast}, \mathbf{arrive}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\} \\
&\text{where } C = \{\mathbf{cast}|\overline{\mathbf{cast}} \rightarrow \mathbf{starcast}, \neg\mathbf{uni}|\overline{\neg\mathbf{uni}} \rightarrow \mathbf{t}, \mathbf{del}|\overline{\mathbf{del}} \rightarrow \mathbf{deliver}, \mathbf{receive}|\overline{\mathbf{receive}} \rightarrow \mathbf{arrive}\} \\
\text{where } \mathrm{G}(\mathtt{ip}, \mathtt{R}) &= \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (\mathtt{R} \cap \mathtt{D} = \mathtt{D'}) \rightarrow \overline{\mathbf{cast}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) \\
&\quad + \textstyle\sum_{\mathtt{d}:\mathrm{IP},\mathtt{msg}:\mathrm{MSG}} (\mathtt{d} \notin \mathtt{R}) \rightarrow \overline{\neg\mathbf{uni}}(\{\mathtt{d}\}, \emptyset, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) \\
&\quad + \textstyle\sum_{\mathtt{data}:\mathrm{DATA}} \overline{\mathbf{del}}(\mathtt{ip}, \mathtt{data}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) \\
&\quad + \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (\mathtt{ip} \in \mathtt{D'}) \rightarrow \overline{\mathbf{receive}}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) \\
&\quad + \textstyle\sum_{\mathtt{D},\mathtt{D'}:\mathrm{Set(IP)},\mathtt{msg}:\mathrm{MSG}} (\mathtt{ip} \notin \mathtt{D'}) \rightarrow \mathbf{arrive}(\mathtt{D}, \mathtt{D'}, \mathtt{msg}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) \\
&\quad + \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(\mathtt{ip}, \mathtt{ip'}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \cup \{\mathtt{ip'}\}) \\
&\quad + \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{connect}(\mathtt{ip'}, \mathtt{ip}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \cup \{\mathtt{ip'}\}) \\
&\quad + \textstyle\sum_{\mathtt{ip'},\mathtt{ip''}:\mathrm{IP}} (\mathtt{ip} \notin \{\mathtt{ip'}, \mathtt{ip''}\}) \rightarrow \mathbf{connect}(\mathtt{ip'}, \mathtt{ip''}).\mathrm{G}(\mathtt{ip}, \mathtt{R}) \\
&\quad + \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{disconnect}(\mathtt{ip}, \mathtt{ip'}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \setminus \{\mathtt{ip'}\}) \\
&\quad + \textstyle\sum_{\mathtt{ip'}:\mathrm{IP}} \mathbf{disconnect}(\mathtt{ip'}, \mathtt{ip}).\mathrm{G}(\mathtt{ip}, \mathtt{R} \setminus \{\mathtt{ip'}\}) \\
&\quad + \textstyle\sum_{\mathtt{ip'},\mathtt{ip''}:\mathrm{IP}} (\mathtt{ip} \notin \{\mathtt{ip'}, \mathtt{ip''}\}) \rightarrow \mathbf{disconnect}(\mathtt{ip'}, \mathtt{ip''}).\mathrm{G}(\mathtt{ip}, \mathtt{R})
\end{aligned}
$$

Process expressions $\mathrm{T}(ip : \mathrm{P} : R)$ produced by this translation rule have by design a ∇_V operator on the outside. As a consequence of mCRL2 inference rule Allow 2, if $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{Q}$ then $a \in V \cup \{\tau\} = \{\tau, \mathbf{starcast}, \mathbf{arrive}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}$. The induction hypothesis must be proven for each of these actions.

1: τ actions.
There are exactly two possible sources for τ actions in T14: hiding $\neg$**unicast** or leaving a τ action performed by $\mathrm{T}(\mathrm{P})$ unchanged and unblocked. A case distinction is made based on the source:

- The first source corresponds with the derivation found in the Lemma 4.6-proof for Unicast (T3-2), which is a representative derivation without alternatives (see Definition 4.9): there are no other derivations that produce a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R)$ in which a $\neg$**unicast** action is hidden.
  The induction hypothesis states that there must be some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, \tau \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}')$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{P}'$ for all $a' \in A_2$. Clearly,

  $$(A_1, A_2) = (\{\ \neg\mathbf{uni}([\![\mathrm{T}_\xi(dest)]\!], [\![\mathrm{T}_\xi(ms)]\!])\ \}, \{\ \neg\mathbf{unicast}(\xi(dest), \xi(ms))\ \})$$

  and therefore $\mathrm{P} \xrightarrow{\neg\mathbf{unicast}(dip,m)} \mathrm{P}'$. Additionally, from the assumption that $[\![t_{\mathrm{U}(dip)} \notin t_{\mathrm{U}(R)}]\!] = \mathit{true}$ in the derivation follows that $dip \notin R$. It then becomes clear that inference rule Unicast (T3-2) can be used to reach the conclusion that $ip : \mathrm{P} : R \xrightarrow{\tau} ip : \mathrm{P}' : R$.
- The second source for τ corresponds with the derivation found in the Lemma 4.6-proof for Internal (T3), which is a representative derivation without alternatives (see Definition 4.9): there are no other derivations that produce a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R)$ in which a τ action performed by $\mathrm{T}(\mathrm{P})$ is left unchanged and unblocked.
  The induction hypothesis states that there must be some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, \tau \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}')$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{P}'$ for all $a' \in A_2$. Clearly, $(A_1, A_2) = (\{\ \tau\ \}, \{\ \tau\ \})$ and therefore $\mathrm{P} \xrightarrow{\tau} \mathrm{P}'$. Inference rule Internal (T3) can then be used to reach the conclusion that $ip : \mathrm{P} : R \xrightarrow{\tau} ip : \mathrm{P}' : R$.

Having covered all possibilities for $\mathrm{T}(ip : \mathrm{P} : R)$ to do a τ action, Lemma 4.8 has been proven for case 1.

2: *Case 2:* **starcast** actions.
In order for $\mathrm{T}(ip : \mathrm{P} : R)$ to produce a **starcast** action, $\mathrm{T}(\mathrm{P})$ must produce a **cast** action that synchronizes with a $\overline{\mathbf{cast}}$ action of G. The induction hypothesis states that – if that is the case – there must be some $(A_1, A_2) \in \mathcal{A}^{\smile} \,.\, \mathbf{cast}(\mathrm{U}(D), \mathrm{U}(R), \mathrm{U}(m)) \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}')$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{P}'$ for

all $a' \in A_2$. These are the candidates for (A_1, A_2):

$$(\{\ \mathbf{cast}(\llbracket \mathcal{U}_{\mathrm{IP}} \rrbracket, \llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket) \mid \hat{\mathrm{D}} : \mathrm{Set(IP)}\ \}, \{\ \mathbf{broadcast}(\xi(ms))\ \})$$
$$(\{\ \mathbf{cast}(\llbracket \mathrm{T}_\xi(dests) \rrbracket, \llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket) \mid \hat{\mathrm{D}} : \mathrm{Set(IP)}\ \}, \{\ \mathbf{groupcast}(\xi(dests), \xi(ms))\ \})$$
$$(\{\ \mathbf{cast}(\llbracket \mathrm{T}_\xi(\{dest\}) \rrbracket, \llbracket \mathrm{T}_\xi(\{dest\}) \rrbracket, \llbracket \mathrm{T}_\xi(ms) \rrbracket)\ \}, \{\ \mathbf{unicast}(\xi(dest), \xi(ms))\ \})$$

In other words, one of the following premises must hold:

$$\mathrm{P} \xrightarrow{\mathbf{broadcast}(\xi(ms))} \mathrm{P}' \quad \text{or} \quad \mathrm{P} \xrightarrow{\mathbf{groupcast}(\xi(dests),\xi(ms))} \mathrm{P}' \quad \text{or} \quad \mathrm{P} \xrightarrow{\mathbf{unicast}(\xi(dest),\xi(ms))} \mathrm{P}'$$

A case distinction is made for these premises:

- Suppose that $\mathrm{P} \xrightarrow{\mathbf{broadcast}(\xi(ms))} \mathrm{P}'$. This leads to a derivation that can be found in the Lemma 4.6-proof for Broadcast (T3). Since no other derivations that are based on the premise *and* that lead to a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{Q}$ are possible (the derivation is a representative derivation without alternatives, see Definition 4.9), AWN must be able to do the transition

  $$ip : \mathrm{P} : R \xrightarrow{R:*\mathbf{cast}(\xi(ms))} ip : \mathrm{P}' : R$$

  for some *ip* in order to mimic mCRL2. Indeed, applying inference rule Broadcast (T3) produces the required conclusion.
- Suppose that $\mathrm{P} \xrightarrow{\mathbf{groupcast}(\xi(dests),\xi(ms))} \mathrm{P}'$. This leads to a derivation that can be found in the Lemma 4.6-proof for Groupcast (T3). Since no other derivations that are based on the premise *and* that lead to a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{Q}$ are possible (the derivation is a representative derivation without alternatives, see Definition 4.9, AWN must be able to do the transition

  $$ip : \mathrm{P} : R \xrightarrow{R \cap D:*\mathbf{cast}(\xi(ms))} ip : \mathrm{P}' : R$$

  for some *ip* in order to mimic mCRL2. Indeed, applying inference rule Groupcast (T3) produces the required conclusion.
- Suppose that $\mathrm{P} \xrightarrow{\mathbf{unicast}(\xi(dest),\xi(ms))} \mathrm{P}'$. This leads to a derivation that can be found in the Lemma 4.6-proof for Unicast (T3-1). Since no other derivations that are based on the premise *and* that lead to a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R) \xrightarrow{a} \mathrm{Q}$ are possible (the derivation is a representative derivation without alternatives, see Definition 4.9, AWN must be able to do the transition

  $$ip : \mathrm{P} : R \xrightarrow{\{dip\}:*\mathbf{cast}(\xi(ms))} ip : \mathrm{P}' : R$$

  for some *ip* in order to mimic mCRL2. Indeed, applying inference rule Unicast (T3-1) produces the required conclusion, but only under the condition that $dip \in R$. This follows from $\llbracket t_{\mathrm{U}(R)} \cap t_{\mathrm{U}(\{dip\})} = t_{\mathrm{U}(\{dip\})} \rrbracket = true$, an assumption that is necessary for the derivation in the Lemma 4.6-proof for Unicast (T3-1).

Having covered all possibilities for $\mathrm{T}(ip : \mathrm{P} : R)$ to do a **starcast** action, Lemma 4.8 has been proven for case 2.

3: *Case 3:* **arrive** actions.
There are exactly two possible sources for **arrive** actions in T14: letting G generate an **arrive** action or converting a **receive** action performed by $\mathrm{T}(\mathrm{P})$. A case distinction is made based on the source:

- The first source corresponds with the derivation found in the Lemma 4.6-proof for Arrive (T3-2), which is a representative derivation without alternatives (see Definition 4.9): there are no other derivations that produce a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R)$ in which G generates an **arrive** action.
  AWN can copy the behavior via inference rule Arrive (T3-2); Lemma 4.8 applies because

  $$(\left\{\ \mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}' \rrbracket, \mathrm{U}(m)) \;\middle|\; \begin{array}{c} \hat{\mathrm{D}}, \hat{\mathrm{D}}' : \mathrm{Set(IP)} \\ \hat{\mathrm{D}}' \subseteq \hat{\mathrm{D}} \\ H \subseteq \bar{\hat{\mathrm{D}}}' \\ K \cap \hat{\mathrm{D}}' = \emptyset \end{array} \right\}, \{\ H \neg K : \mathbf{arrive}(m)\ \})$$

  can be chosen as $(A_1, A_2) \in \mathcal{A}^{\smile}$. $\mathbf{arrive}(\llbracket \hat{\mathrm{D}} \rrbracket, \llbracket \hat{\mathrm{D}}' \rrbracket, \mathrm{U}(m)) \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}')$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{P}'$ for all $a' \in A_2$.

- The second source for **arrive** corresponds with the derivation found in the Lemma 4.6-proof for Arrive (T3-1), which is a representative derivation without alternatives (see Definition 4.9): there are no other derivations that produce a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R)$ in which a **receive** action is converted.
  In this particular case, the induction hypothesis states that there must be some $(A_1, A_2) \in \mathcal{A}^{\smile}$. $\mathbf{receive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathrm{U}(m)) \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}')$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{P}'$ for all $a' \in A_2$. Clearly,

$$(\{\ \mathbf{receive}(\llbracket \hat{\mathtt{D}} \rrbracket, \llbracket \hat{\mathtt{D}}\mathtt{'} \rrbracket, \mathrm{U}(m)) \mid \hat{\mathtt{D}}, \hat{\mathtt{D}}\mathtt{'} : \mathrm{Set}(\mathrm{IP})\ \}, \{\ \mathbf{receive}(m)\ \})$$

  and therefore $\mathrm{P} \xrightarrow{\mathbf{receive}(m)} \mathrm{P}'$. Inference rule ARRIVE (T3-1) can then be used to reach the conclusion that $ip : \mathrm{P} : R \xrightarrow{\{ip\} \neg \emptyset : \mathbf{arrive}(m)} ip : \mathrm{P}' : R$.

Having covered all possibilities for $\mathrm{T}(ip : \mathrm{P} : R)$ to do an **arrive** action, Lemma 4.8 has been proven for case 3.

4: *Case 4:* **deliver** actions.
In order for $\mathrm{T}(ip : \mathrm{P} : R)$ to produce a **deliver** action, $\mathrm{T}(\mathrm{P})$ must produce a **del** action that synchronizes with a $\overline{\mathbf{del}}$ action of G. This corresponds with the derivation found in the Lemma 4.6-proof for Deliver (T3), which is a representative derivation without alternatives (see Definition 4.9): there are no other derivations that produce a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R)$ in which **del** and $\overline{\mathbf{del}}$ are synchronized.
The induction hypothesis states that – if that is the case – there must be some $(A_1, A_2) \in \mathcal{A}^{\smile}$. $\mathbf{del}(\mathrm{U}(ip), \mathrm{U}(d)) \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}')$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{P}'$ for all $a' \in A_2$. There is only one candidate for (A_1, A_2), namely

$$(\{\ \mathbf{del}(\llbracket \hat{\mathtt{i}} \rrbracket, \llbracket \mathrm{T}_{\xi}(data) \rrbracket) \mid \hat{\mathtt{i}} : \mathrm{IP}\ \}, \{\ \mathbf{deliver}(\xi(data))\ \})$$

It follows that $\mathrm{P} \xrightarrow{\mathbf{deliver}(\xi(data))} \mathrm{P}'$, which can serve as the premise for inference rule DELIVER (T3) to prove that $ip : \mathrm{P} : R \xrightarrow{ip : \mathbf{deliver}(\xi(data))} ip : \mathrm{P}' : R$. This covers all possibilities for $\mathrm{T}(ip : \mathrm{P} : R)$ to do a **deliver** action, thus proving Lemma 4.8 for case 4.

5: *Case 5:* **connect** actions.
Process expressions $\mathrm{T}(ip : \mathrm{P} : R)$ can only produce **connect** actions if they are generated by G such as in the derivation found in the Lemma 4.6-proof for Connect (T3-1). This derivation as well as the comparable derivations from the proofs for CONNECT (T3-2) and CONNECT (T3-3) are representative derivations without alternatives (see Definition 4.9): there are no other derivations that produce a conclusion of the form $\mathrm{T}(ip : \mathrm{P} : R)$ in which G generates a **connect** action.
AWN can copy the behavior of mCRL2 via inference rules CONNECT (T3-1), CONNECT (T3-2), and CONNECT (T3-3); Lemma 4.8 applies because

$$(\{\ \mathbf{connect}(\mathrm{U}(ip\text{'}), \mathrm{U}(ip\text{''}))\ \}, \{\ \mathbf{connect}(ip\text{'}, ip\text{''})\ \})$$

can be chosen as $(A_1, A_2) \in \mathcal{A}^{\smile}$. $\mathbf{connect}(\mathrm{U}(ip), \mathrm{U}(ip\text{'})) \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}')$ such that $\mathrm{P} \xrightarrow{a'} \mathrm{P}'$ for all $a' \in A_2$.
This covers all possibilities for $\mathrm{T}(ip : \mathrm{P} : R)$ to do a **connect** action, thus proving Lemma 4.8 for case 5.

6: *Case 6:* **disconnect** actions.
Similar to the proof for Case 5.

Translation rule T15: The translation function T is partially defined by translation rule

$$\begin{aligned}&\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) = \rho_R \nabla_V \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\rightarrow\mathbf{a}\}} \Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \quad \text{T15}\\ &\quad \text{where } R = \{\mathbf{a} \rightarrow \mathbf{arrive}, \mathbf{c} \rightarrow \mathbf{connect}, \mathbf{d} \rightarrow \mathbf{disconnect}, \mathbf{s} \rightarrow \mathbf{starcast}\}\\ &\quad \text{where } V = \{\mathbf{a}, \mathbf{c}, \mathbf{d}, \mathbf{deliver}, \mathbf{s}, \mathbf{t}\}\\ &\quad \text{where } C = \{\mathbf{starcast}|\mathbf{arrive} \rightarrow \mathbf{s}, \mathbf{connect}|\mathbf{connect} \rightarrow \mathbf{c}, \mathbf{disconnect}|\mathbf{disconnect} \rightarrow \mathbf{d}\}\end{aligned}$$

Consider several pairs from Table 2.6:

$$\begin{aligned}(\{\ \tau\ \}, \{\ \mathbf{t}(\mathrm{u}(D), \mathrm{u}(R), \mathrm{u}(m))\ \})\\ (\{\ R{:}{*}\mathbf{cast}(m)\ \}, \{\ \mathbf{starcast}(\mathrm{u}(D), \mathrm{u}(R), \mathrm{u}(m))\ \})\\ (\{\ ip{:}\mathbf{deliver}(d)\ \}, \{\ \mathbf{deliver}(\mathrm{u}(ip), \mathrm{u}(d))\ \})\\ (\{\ H\neg K{:}\mathbf{arrive}(m)\ \}, \left\{\ \mathbf{arrive}([\![\hat{\mathrm{D}}]\!], [\![\hat{\mathrm{D}}\text{'}]\!], \mathrm{u}(m)) \;\middle|\; \begin{smallmatrix}\hat{\mathrm{D}},\hat{\mathrm{D}}\text{'}:\mathrm{Set(IP)}\\ \hat{\mathrm{D}}\text{'}\subseteq\hat{\mathrm{D}}\\ H\subseteq\hat{\mathrm{D}}\text{'}\\ K\cap\hat{\mathrm{D}}\text{'}=\emptyset\end{smallmatrix} \right\})\\ (\{\ \mathbf{connect}(ip\text{'}, ip\text{''})\ \}, \{\ \mathbf{connect}(\mathrm{u}(ip\text{'}), \mathrm{u}(ip\text{''}))\ \})\\ (\{\ \mathbf{disconnect}(ip\text{'}, ip\text{''})\ \}, \{\ \mathbf{disconnect}(\mathrm{u}(ip\text{'}), \mathrm{u}(ip\text{''}))\ \})\end{aligned}$$

Let T_1, S_1, L_1, A_1, C_1, and D_1 be defined as the second members of these pairs; that is, let

$$\begin{aligned}T_1 &\stackrel{\text{def}}{=} \{\ \mathbf{t}(\mathrm{u}(D), \mathrm{u}(R), \mathrm{u}(m))\ \})\\ S_1 &\stackrel{\text{def}}{=} \{\ \mathbf{starcast}(\mathrm{u}(D), \mathrm{u}(R), \mathrm{u}(m))\ \}\\ L_1 &\stackrel{\text{def}}{=} \{\ \mathbf{deliver}(\mathrm{u}(ip), \mathrm{u}(d))\ \}\\ A_1 &\stackrel{\text{def}}{=} \left\{\ \mathbf{arrive}([\![\hat{\mathrm{D}}]\!], [\![\hat{\mathrm{D}}\text{'}]\!], \mathrm{u}(m)) \;\middle|\; \begin{smallmatrix}\hat{\mathrm{D}},\hat{\mathrm{D}}\text{'}:\mathrm{Set(IP)}\\ \hat{\mathrm{D}}\text{'}\subseteq\hat{\mathrm{D}}\\ H\subseteq\hat{\mathrm{D}}\text{'}\\ K\cap\hat{\mathrm{D}}\text{'}=\emptyset\end{smallmatrix} \right\})\\ C_1 &\stackrel{\text{def}}{=} \{\ \mathbf{connect}(\mathrm{u}(ip\text{'}), \mathrm{u}(ip\text{''}))\ \}\\ D_1 &\stackrel{\text{def}}{=} \{\ \mathbf{disconnect}(\mathrm{u}(ip\text{'}), \mathrm{u}(ip\text{''}))\ \}\end{aligned}$$

for some d : DATA, ip, ip', ip'' : IP, D, R, H, K : Set(IP), and m : MSG.
The following cases are distinguished:

1: $\mathrm{T}(\mathrm{M})$ or $\mathrm{T}(\mathrm{N})$ does a transition $a \in T_1$; the other process does not do a transition.
2: $\mathrm{T}(\mathrm{M})$ or $\mathrm{T}(\mathrm{N})$ does a transition $a \in L_1$; the other process does not do a transition.
3: $\mathrm{T}(\mathrm{M})$ or $\mathrm{T}(\mathrm{N})$ does a transition a where $\underline{a} \notin \{\mathbf{t}, \mathbf{deliver}\}$; the other process does not do a transition.
4: $\mathrm{T}(\mathrm{M})$ and $\mathrm{T}(\mathrm{N})$ both do the same transition $a \in A_1$.
5: $\mathrm{T}(\mathrm{M})$ and $\mathrm{T}(\mathrm{N})$ both do the same transition $a \in C_1$.
6: $\mathrm{T}(\mathrm{M})$ and $\mathrm{T}(\mathrm{N})$ both do the same transition $a \in D_1$.
7: $\mathrm{T}(\mathrm{M})$ and $\mathrm{T}(\mathrm{N})$ both do the same transition a where $\underline{a} \notin \{\mathbf{arrive}, \mathbf{connect}, \mathbf{disconnect}\}$.
8: $\mathrm{T}(\mathrm{M})$ does a transition $a \in S_1$ and $\mathrm{T}(\mathrm{N})$ does a transition $b \in A_1$, or vice versa.
9: $\mathrm{T}(\mathrm{M})$ does a transition $a \in S_1$ and $\mathrm{T}(\mathrm{N})$ does a transition $b \notin A_1$ where $\underline{b} = \mathbf{arrive}$, or vice versa.
10: $\mathrm{T}(\mathrm{M})$ does a transition a and $\mathrm{T}(\mathrm{N})$ does a transition $b \neq a$ where $\{\underline{a}, \underline{b}\} \neq \{\mathbf{starcast}, \mathbf{arrive}\}$.

Note that these cases cover all combinations of behavior of $\mathrm{T}(\mathrm{M})$ and $\mathrm{T}(\mathrm{N})$.
The proof is provided below for each of the cases:

1: Suppose that $\mathrm{T}(\mathrm{M})$ or $\mathrm{T}(\mathrm{N})$ does a transition $a \in T_1$; the other process does not do a transition. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from T_1, it can first be concluded that

$$\mathrm{T}(\mathrm{M}) \xrightarrow{T_1} \mathrm{T}(\mathrm{M}') \wedge \mathrm{M} \xrightarrow{\tau} \mathrm{M}'$$

The mCRL2 derivation below shows how the first conjunct is sufficient to prove that $\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) \xrightarrow{T_1} \mathrm{T}(\mathrm{M}' \mid\mid \mathrm{N})$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T(M)} \xrightarrow{\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \mathrm{T(M')}}\text{ Induction hypothesis}}{\mathrm{T(M)} \mid\mid \mathrm{T(N)} \xrightarrow{\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \mathrm{T(M')} \mid\mid \mathrm{T(N)}}\text{ Par 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\gamma_C(\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m)))} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N)})}\text{ Comm 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N)})}\text{ Apply }\gamma_C}{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}(\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m)))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N)})}\text{ Comm 2}}{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N)})}\text{ Apply }\gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}}{\tau\in V\cup\{\tau\}\quad \nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N)})}\text{ Allow 2}}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{R\bullet\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N)})}\text{ Rename 2}}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)}) \xrightarrow{\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{T(N)})}\text{ Apply }R\bullet}{\mathrm{T(M} \mid\mid \mathrm{N)} \xrightarrow{\mathbf{t}(\mathrm{u}(D),\mathrm{u}(R),\mathrm{u}(m))} \mathrm{T(M'} \mid\mid \mathrm{N)}}\text{ T15}$$

On the AWN side, the second conjunct can be used as premise in

$$\dfrac{\mathrm{M} \xrightarrow{\tau} \mathrm{M'}}{\mathrm{M} \mid\mid \mathrm{N} \xrightarrow{\tau} \mathrm{M'} \mid\mid \mathrm{N}}\text{ Internal (T4-1)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $\mathbf{t}(\mathrm{u}(D), \mathrm{u}(R), \mathrm{u}(m)) \in A_1 \wedge \mathrm{T(M} \mid\mid \mathrm{N)} \xrightarrow{A_1} \mathrm{T(M'} \mid\mid \mathrm{N)} \wedge \mathrm{M} \mid\mid \mathrm{N} \xrightarrow{A_2} \mathrm{M'} \mid\mid \mathrm{N}$, and the converse of Pair 2.3 satisfies this requirement.
The proof for when N does the **t** action is similar.

2: Suppose that $\mathrm{T(M)}$ or $\mathrm{T(N)}$ does a transition $a \in L_1$; the other process does not do a transition. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from L_1, it can first be concluded that

$$\mathrm{T(M)} \xrightarrow{L_1} \mathrm{T(M')} \wedge \mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M'}$$

The mCRL2 derivation in the Lemma 4.6-proof for Deliver (T4-1) shows how the first conjunct is sufficient to prove that $\mathrm{T(M} \mid\mid \mathrm{N)} \xrightarrow{L_1} \mathrm{T(M'} \mid\mid \mathrm{N)}$.
On the AWN side, the second conjunct can be used as premise in

$$\dfrac{\mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M'}}{\mathrm{M} \mid\mid \mathrm{N} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M'} \mid\mid \mathrm{N}}\text{ Deliver (T4-1)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T(M} \mid\mid \mathrm{N)} \xrightarrow{A_1} \mathrm{T(M'} \mid\mid \mathrm{N)} \wedge \mathrm{M} \mid\mid \mathrm{N} \xrightarrow{A_2} \mathrm{M'} \mid\mid \mathrm{N}$, and the converse of Pair 2.12 satisfies this requirement.
The proof for when N does the **deliver** action is similar.

3: Suppose that $\mathrm{T(M)}$ or $\mathrm{T(N)}$ does a transition a where $\underline{a} \notin \{\mathbf{t}, \mathbf{deliver}\}$; the other process does not do a transition.
This situation fails to generate behavior for $\mathrm{T(M} \mid\mid \mathrm{N)}$ because no derivation similar to the one in the Lemma 4.6-proof for Deliver (T4-1) is possible:

a. The condition $\underline{a} \notin \{\mathbf{t}, \mathbf{deliver}\}$ implies that

$$\underline{a} \subseteq \{\mathbf{cast}, \neg\mathbf{uni}, \mathbf{receive}, \mathbf{send}, \mathbf{del}, \mathbf{starcast}, \mathbf{arrive}, \mathbf{connect}, \mathbf{disconnect}, \mathbf{newpkt}\}$$

because these are all action labels from the right-hand side of the action relation $\mathcal{A}$;

b. $\mathrm{T(M)} \mid\mid \mathrm{T(N)}$ does not produce multi-actions because only one of $\mathrm{T(M)}$ and $\mathrm{T(N)}$ does a transition;

c. The communication operator Γ_C has no effect because $\mathrm{T(M)} \mid\mid \mathrm{T(N)}$ does not produce multi-actions, and therefore no **c**, **d**, or **s** actions are generated;

d. The communication operator $\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}$ has no effect because $\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)})$ does not produce multi-actions, and therefore no **a** action is generated;

e. The allow operator ∇_V blocks all actions except **a**, **c**, **d**, **deliver**, **s**, or **t**, none of which can be produced by $\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{T(N)})$.

Since $\mathrm{T(M)} \mid\mid \mathrm{T(N)}$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

4: Suppose that $\mathrm{T(M)}$ and $\mathrm{T(N)}$ both do the same transition $a \in A_1$. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels

from A_1, it can first be concluded that

$$\mathrm{T(M)} \xrightarrow{A_1} \mathrm{T(M')} \wedge \mathrm{M} \xrightarrow{H\neg K:\mathbf{arrive}(m)} \mathrm{M'} \wedge \mathrm{T(N)} \xrightarrow{A_1} \mathrm{T(N')} \wedge \mathrm{N} \xrightarrow{H'\neg K':\mathbf{arrive}(m)} \mathrm{N'}$$

The mCRL2 derivation in the Lemma 4.6-proof for Cast (T4-3) shows how the first and third conjunct are sufficient to prove that $\mathrm{T(M \mid\mid N)} \xrightarrow{A_1} \mathrm{T(M' \mid\mid N)}$.
On the AWN side, the second and fourth conjunct can be used as premise in

$$\frac{\mathrm{M} \xrightarrow{H\neg K:\mathbf{arrive}(m)} \mathrm{M'} \qquad \mathrm{N} \xrightarrow{H'\neg K':\mathbf{arrive}(m)} \mathrm{N'}}{\mathrm{M \mid\mid N} \xrightarrow{(H\cup H')\neg(K\cup K'):\mathbf{arrive}(m)} \mathrm{M' \mid\mid N'}} \ \textsc{Cast (T4-3)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T(M \mid\mid N)} \xrightarrow{A_1} \mathrm{T(M' \mid\mid N)} \wedge \mathrm{M \mid\mid N} \xrightarrow{A_2} \mathrm{M' \mid\mid N}$, and the converse of Pair 2.13 satisfies this requirement.

5: Suppose that $\mathrm{T(M)}$ and $\mathrm{T(N)}$ both do the same transition $a \in C_1$. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from C_1, it can first be concluded that

$$\mathrm{T(M)} \xrightarrow{C_1} \mathrm{T(M')} \wedge \mathrm{M} \xrightarrow{\mathbf{connect}(ip',ip'')} \mathrm{M'} \wedge \mathrm{T(N)} \xrightarrow{C_1} \mathrm{T(N')} \wedge \mathrm{N} \xrightarrow{\mathbf{connect}(ip',ip'')} \mathrm{N'}$$

The mCRL2 derivation in the Lemma 4.6-proof for Connect (T4-1) shows how the first and third conjunct are sufficient to prove that $\mathrm{T(M \mid\mid N)} \xrightarrow{C_1} \mathrm{T(M' \mid\mid N)}$.
On the AWN side, the second and fourth conjunct can be used as premise in

$$\frac{\mathrm{M} \xrightarrow{\mathbf{connect}(ip',ip'')} \mathrm{M'} \qquad \mathrm{N} \xrightarrow{\mathbf{connect}(ip',ip'')} \mathrm{N'}}{\mathrm{M \mid\mid N} \xrightarrow{\mathbf{connect}(ip',ip'')} \mathrm{M' \mid\mid N'}} \ \textsc{Connect (T4-1)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T(M \mid\mid N)} \xrightarrow{A_1} \mathrm{T(M' \mid\mid N)} \wedge \mathrm{M \mid\mid N} \xrightarrow{A_2} \mathrm{M' \mid\mid N}$, and the converse of Pair 2.14 satisfies this requirement.

6: Suppose that $\mathrm{T(M)}$ and $\mathrm{T(N)}$ both do the same transition $a \in D_1$. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from D_1, it can first be concluded that

$$\mathrm{T(M)} \xrightarrow{D_1} \mathrm{T(M')} \wedge \mathrm{M} \xrightarrow{\mathbf{disconnect}(ip',ip'')} \mathrm{M'} \wedge \mathrm{T(N)} \xrightarrow{D_1} \mathrm{T(N')} \wedge \mathrm{N} \xrightarrow{\mathbf{disconnect}(ip',ip'')} \mathrm{N'}$$

The mCRL2 derivation below shows how the first and third conjunct are sufficient to prove that $\mathrm{T(M \mid\mid N)} \xrightarrow{D_1} \mathrm{T(M' \mid\mid N)}$:

$$\frac{}{\mathrm{T(M)} \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \mathrm{T(M')}} \ \text{Induction hypothesis} \qquad \frac{}{\mathrm{T(N)} \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \mathrm{T(N')}} \ \text{Induction hypothesis}$$

$$\frac{\mathrm{T(M)} \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \mathrm{T(M')} \qquad \mathrm{T(N)} \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \mathrm{T(N')}}{\mathrm{T(M) \mid\mid T(N)} \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))|\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \mathrm{T(M') \mid\mid T(N')}} \ \textsc{Par 3}$$

$$\frac{\mathrm{T(M) \mid\mid T(N)} \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))|\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \mathrm{T(M') \mid\mid T(N')}}{\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\gamma_C(\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))|\mathbf{disconnect}(\upsilon(ip),\upsilon(ip')))} \Gamma_C(\mathrm{T(M') \mid\mid T(N')})} \ \textsc{Comm 2}$$

$$\frac{\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\gamma_C(\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))|\mathbf{disconnect}(\upsilon(ip),\upsilon(ip')))} \Gamma_C(\mathrm{T(M') \mid\mid T(N')})}{\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \Gamma_C(\mathrm{T(M') \mid\mid T(N')})} \ \text{Apply } \gamma_C$$

$$\frac{\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \Gamma_C(\mathrm{T(M') \mid\mid T(N')})}{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}(\mathbf{d}(\upsilon(ip),\upsilon(ip')))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})} \ \textsc{Comm 2}$$

$$\frac{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}(\mathbf{d}(\upsilon(ip),\upsilon(ip')))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})}{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})} \ \text{Apply } \gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}$$

$$\mathbf{d} \underline{\in} V \cup \{\tau\} \ \frac{\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})}{\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})} \ \textsc{Allow 2}$$

$$\frac{\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{R\bullet\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})} \ \textsc{Rename 2}$$

$$\frac{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{R\bullet\mathbf{d}(\upsilon(ip),\upsilon(ip'))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})}{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})} \ \text{Apply } R\bullet$$

$$\frac{\rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M) \mid\mid T(N)}) \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \rho_R\nabla_V\Gamma_{\{\mathbf{arrive}|\mathbf{arrive}\to\mathbf{a}\}}\Gamma_C(\mathrm{T(M') \mid\mid T(N')})}{\mathrm{T(M \mid\mid N)} \xrightarrow{\mathbf{disconnect}(\upsilon(ip),\upsilon(ip'))} \mathrm{T(M' \mid\mid N')}} \ \text{T15}$$

On the AWN side, the second and fourth conjunct can be used as premise in

$$\frac{\mathrm{M} \xrightarrow{\mathbf{disconnect}(ip',ip'')} \mathrm{M'} \qquad \mathrm{N} \xrightarrow{\mathbf{disconnect}(ip',ip'')} \mathrm{N'}}{\mathrm{M \mid\mid N} \xrightarrow{\mathbf{disconnect}(ip',ip'')} \mathrm{M' \mid\mid N'}} \ \textsc{Disconnect (T4-1)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T(M \mid\mid N)} \xrightarrow{A_1} \mathrm{T(M' \mid\mid N)} \wedge \mathrm{M \mid\mid N} \xrightarrow{A_2} \mathrm{M' \mid\mid N}$, and the converse of Pair 2.15 satisfies this requirement.

7: Suppose that $\mathrm{T}(\mathrm{M})$ and $\mathrm{T}(\mathrm{N})$ both do the same transition a where $\underline{a} \notin \{\textbf{arrive}, \textbf{connect}, \textbf{disconnect}\}$. The mCRL2 derivation below shows where the attempt to generate behavior for $\mathrm{T}(\mathrm{P} \lll \mathrm{Q})$ under these circumstances fails:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{a} \mathrm{T}(\mathrm{M}')}\text{ Induction hypothesis} \quad \dfrac{}{\mathrm{T}(\mathrm{N}) \xrightarrow{a} \mathrm{T}(\mathrm{N}')}\text{ Induction hypothesis}}{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N}) \xrightarrow{a|a} \mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}')}\text{ Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_C(a|a)} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{a|a} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Apply } \gamma_C}{\Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}(a|a)} \Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Comm 2}}{\Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{a|a} \Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Apply } \gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}$$

The Allow 2 operator cannot be applied next (like in Case 6) because $\underline{a|a} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N})$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

8: Suppose that $\mathrm{T}(\mathrm{M})$ does a transition $a \in S_1$ and $\mathrm{T}(\mathrm{N})$ does a transition $b \in A_1$, or vice versa. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from S_1 or A_1, it can first be concluded that

$$\mathrm{T}(\mathrm{M}) \xrightarrow{S_1} \mathrm{T}(\mathrm{M}') \wedge \mathrm{M} \xrightarrow{R:*\textbf{cast}(m)} \mathrm{M}' \wedge \mathrm{T}(\mathrm{N}) \xrightarrow{A_1} \mathrm{T}(\mathrm{N}') \wedge \mathrm{N} \xrightarrow{H\neg K:\textbf{arrive}(m)} \mathrm{N}'$$

The mCRL2 derivation in the Lemma 4.6-proof for Cast (T4-1) shows how the first and third conjunct are sufficient to prove that $\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) \xrightarrow{S_1} \mathrm{T}(\mathrm{M}' \mid\mid \mathrm{N})$.
On the AWN side, the second and fourth conjunct can be used as premise in

$$H \subseteq R \wedge K \cap R = \emptyset \;\dfrac{\mathrm{M} \xrightarrow{R:*\textbf{cast}(m)} \mathrm{M}' \qquad \mathrm{N} \xrightarrow{H\neg K:\textbf{arrive}(m)} \mathrm{N}'}{\mathrm{M} \mid\mid \mathrm{N} \xrightarrow{R:*\textbf{cast}(m)} \mathrm{M}' \mid\mid \mathrm{N}'}\text{ Cast (T4-1)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T}(\mathrm{M} \mid\mid \mathrm{N}) \xrightarrow{A_1} \mathrm{T}(\mathrm{M}' \mid\mid \mathrm{N}) \wedge \mathrm{M} \mid\mid \mathrm{N} \xrightarrow{A_2} \mathrm{M}' \mid\mid \mathrm{N}$, and the converse of Pair 2.11 satisfies this requirement.
The proof for when $\mathrm{T}(\mathrm{M})$ does the **arrive** action and $\mathrm{T}(\mathrm{N})$ does the **starcast** action is similar.

9: Suppose that $\mathrm{T}(\mathrm{M})$ does a transition $a \in S_1$ and $\mathrm{T}(\mathrm{N})$ does a transition $b \notin A_1$ where $\underline{b} = \textbf{arrive}$, or vice versa. The mCRL2 derivation below shows where the attempt to generate behavior for $\mathrm{T}(\mathrm{P} \lll \mathrm{Q})$ under these circumstances fails:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T}(\mathrm{M}) \xrightarrow{a} \mathrm{T}(\mathrm{M}')}\text{ Induction hypothesis} \quad \dfrac{}{\mathrm{T}(\mathrm{N}) \xrightarrow{b} \mathrm{T}(\mathrm{N}')}\text{ Induction hypothesis}}{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N}) \xrightarrow{a|b} \mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}')}\text{ Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_C(a|b)} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{a|b} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Apply } \gamma_C}{\Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{\gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}(a|b)} \Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Comm 2}}{\Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{T}(\mathrm{N})) \xrightarrow{a|b} \Gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{T}(\mathrm{N}'))}\text{ Apply } \gamma_{\{\textbf{arrive}|\textbf{arrive}\to\textbf{a}\}}$$

The Allow 2 operator cannot be applied next (like in Case 6) because $\underline{a|b} \notin V \cup \{\tau\}$. Note that the communication operator Γ_C has no effect because a and b have different arguments.
In conclusion, $\mathrm{T}([\ \mathrm{M}\ ])$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, and therefore there is no behavior for AWN to mimic.

10: Suppose that $\mathrm{T}(\mathrm{M})$ does a transition a and $\mathrm{T}(\mathrm{N})$ does a transition $b \neq a$ where $\{\underline{a}, \underline{b}\} \neq \{\textbf{starcast}, \textbf{arrive}\}$. Generating behavior for $\mathrm{T}(\mathrm{P} \lll \mathrm{Q})$ under these circumstances fails for a similar reason as in Case 9: the communication operator Γ_C has no effect because $a \neq b$ (rather than that just their arguments are different) and therefore it is not possible to eventually apply the Allow 2 operator.

The derivations in mCRL2 from the cases listed above are representative derivations that collectively have no alternatives (see Definition 4.9): there are no other derivations with different or differently ordered

one-way steps that yield different behavior of process expression $\mathrm{T}(\mathrm{M} \mid\mid \mathrm{N})$. Therefore the induction hypothesis generally holds for this expression.

Translation rule T16: The translation function T is partially defined by translation rule

$$\begin{aligned}
&\mathrm{T}([\ \mathrm{M}\ ]) = \nabla_V \rho_{\{\mathbf{starcast}\to\mathbf{t}\}} \Gamma_C(\mathrm{T}(\mathrm{M}) \parallel \mathrm{H}) \quad \text{T16}\\
&\text{where } V = \{\mathbf{t}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}\\
&\text{where } C = \{\overline{\mathbf{newpkt}}|\mathbf{arrive} \to \mathbf{newpkt}\}\\
&\text{where } \mathrm{H} = \textstyle\sum_{\mathtt{ip:IP},\mathtt{data:DATA},\mathtt{dest:IP}} \overline{\mathbf{newpkt}}(\{\mathtt{ip}\},\{\mathtt{ip}\},\mathtt{newpkt}(\mathtt{data},\mathtt{dest})).\mathrm{H}
\end{aligned}$$

Consider two pairs from Table 2.6:

$$(\{\ H\neg K\!:\!\mathbf{arrive}(m)\ \}, \left\{\ \mathbf{arrive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\mathtt{'}]\!],\mathtt{U}(m))\ \middle|\ \begin{smallmatrix}\hat{\mathtt{D}},\hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)}\\ \hat{\mathtt{D}}\mathtt{'}\subseteq\hat{\mathtt{D}}\\ H\subseteq\hat{\mathtt{D}}\mathtt{'}\\ K\cap\hat{\mathtt{D}}\mathtt{'}=\emptyset\end{smallmatrix}\ \right\}),$$
$$(\{\ ip\!:\!\mathbf{newpkt}(d,dip)\ \}, \{\ \mathbf{newpkt}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))\ \})$$

Let A_1 and N_1 be defined as the second members of these pairs; that is, let

$$A_1 \stackrel{\text{def}}{=} \left\{\ \mathbf{arrive}([\![\hat{\mathtt{D}}]\!],[\![\hat{\mathtt{D}}\mathtt{'}]\!],\mathtt{U}(m))\ \middle|\ \begin{smallmatrix}\hat{\mathtt{D}},\hat{\mathtt{D}}\mathtt{'}:\mathrm{Set(IP)}\\ \hat{\mathtt{D}}\mathtt{'}\subseteq\hat{\mathtt{D}}\\ H\subseteq\hat{\mathtt{D}}\mathtt{'}\\ K\cap\hat{\mathtt{D}}\mathtt{'}=\emptyset\end{smallmatrix}\ \right\}$$
$$N_1 \stackrel{\text{def}}{=} \{\ \mathbf{newpkt}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))\ \}$$

and let

$$\overline{N}_1 \stackrel{\text{def}}{=} \left\{\ \overline{\mathbf{newpkt}}(\{\mathtt{U}(ip)\},\{\mathtt{U}(ip)\},\mathtt{newpkt}(\mathtt{U}(d),\mathtt{U}(dip)))\ \right\}$$

for some d : DATA, ip : IP, H,K : Set(IP), and m : MSG.
The following cases are distinguished:

1: $\mathrm{T}(\mathrm{M})$ does a transition $a \in A_1$ and H does a transition $b \in \overline{N}_1$.
2: $\mathrm{T}(\mathrm{M})$ does a transition $a \notin A_1$ and H does a transition $b \in \overline{N}_1$.
3: $\mathrm{T}(\mathrm{M})$ does nothing and H does a transition $b \in \overline{N}_1$.
4: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} = \mathbf{starcast}$ and H does nothing.
5: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} \in \{\mathbf{t}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}$ and H does nothing.
6: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} \notin \{\mathbf{t}, \mathbf{starcast}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}$ and H does nothing.

Note that these cases cover all combinations of behavior of $\mathrm{T}(\mathrm{M})$ and H (H can only do $\overline{\mathbf{newpkt}}$ actions; see the first part of the derivation in the Lemma 4.6-proof for Newpkt (T4), which is a representative derivation without alternatives).
The proof is provided below for each of the cases:

1: $\mathrm{T}(\mathrm{M})$ does a transition $a \in A_1$ and H does a transition $b \in \overline{N}_1$. Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition labels from A_1, it can first be concluded that

$$\mathrm{T}(\mathrm{M}) \xrightarrow{L_1} \mathrm{T}(\mathrm{M}') \wedge \mathrm{M} \xrightarrow{H\neg K:\mathbf{arrive}(m)} \mathrm{M}'$$

The mCRL2 derivation in the Lemma 4.6-proof for Newpkt (T4) shows how the first conjunct is sufficient to prove that $\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{L_1} \mathrm{T}([\ \mathrm{M}'\ ])$. Under the constraints of this case, the derivation is a representative derivation without alternatives (see Definition 4.9) and so all related behavior of $\mathrm{T}([\ \mathrm{M}\ ])$ is covered.
On the AWN side, the second conjunct can be used as premise in

$$\frac{\mathrm{M} \xrightarrow{\{ip\}\neg K:\mathbf{arrive}(\mathtt{newpkt}(d,dip))} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{ip:\mathbf{newpkt}(d,dip)} [\ \mathrm{M}'\ ]}\ \textsc{Newpkt (T4)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{A_1} \mathrm{T}([\ \mathrm{M}'\ ]) \wedge [\ \mathrm{M}\ ] \xrightarrow{A_2} [\ \mathrm{M}'\ ]$, and the converse of Pair 2.16 satisfies this requirement.

2: $\mathrm{T}(\mathrm{M})$ does a transition $a \notin A_1$ and H does a transition $b \in \overline{N}_1$.
This situation fails to generate behavior for $\mathrm{T}([\ \mathrm{M}\ ])$ because no derivation similar to the one in the Lemma 4.6-proof for Newpkt (T4) is possible:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{T}(\mathrm{M}) \xrightarrow{a} \mathrm{T}(\mathrm{M}') \qquad \mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{H}}{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H} \xrightarrow{a|\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H}}\ \textsc{Par 3}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\gamma_C(a|\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip))))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{a|\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet a|\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \textsc{Rename 2}}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{a|\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}') \mid\mid \mathrm{H})}\ \text{Apply } \{\mathbf{starcast}\to\mathbf{t}\}\bullet$$

The Allow 2 operator cannot be applied next because $\underline{a}|\overline{\mathbf{newpkt}} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}([\ \mathrm{M}\ ])$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

3: $\mathrm{T}(\mathrm{M})$ does nothing and H does a transition $\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip))) \in \overline{N}_1$.
This situation fails to generate behavior for $\mathrm{T}([\ \mathrm{M}\ ])$ because no derivation similar to the one in the Lemma 4.6-proof for Newpkt (T4) is possible:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{H}}{\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H} \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}}\ \textsc{Par 5}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\gamma_C(\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip))))} \Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H})}\ \textsc{Comm 2}}{\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H})}\ \text{Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H})}\ \textsc{Rename 2}}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H}) \xrightarrow{\overline{\mathbf{newpkt}}(\{\mathtt{u}(ip)\},\{\mathtt{u}(ip)\},\mathtt{newpkt}(\mathtt{u}(d),\mathtt{u}(dip)))} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T}(\mathrm{M}) \mid\mid \mathrm{H})}\ \text{Apply } \{\mathbf{starcast}\to\mathbf{t}\}\bullet$$

The Allow 2 operator cannot be applied next because $\overline{\mathbf{newpkt}} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}([\ \mathrm{M}\ ])$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.

4: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} = \mathbf{starcast}$ and H does nothing.
Following the induction hypothesis and eliminating pairs from the action relation $\mathcal{A}$ in Table 2.6 that do not match the transition label **starcast**, it can first be concluded that

$$\mathrm{T}(\mathrm{M}) \xrightarrow{\mathbf{starcast}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T}(\mathrm{M}') \wedge \mathrm{M} \xrightarrow{R^*\mathbf{cast}(m)} \mathrm{M}'$$

for all D, R : Set(IP) and m : MSG where $R \subseteq D$.
The mCRL2 derivation in the Lemma 4.6-proof for Cast (T4-4) shows how the first conjunct is sufficient to prove that $\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{\mathbf{t}(\mathtt{u}(D),\mathtt{u}(R),\mathtt{u}(m))} \mathrm{T}([\ \mathrm{M}'\ ])$. Under the constraints of this case, the derivation is a representative derivation without alternatives (see Definition 4.9) and so all related behavior of $\mathrm{T}([\ \mathrm{M}\ ])$ is covered.
On the AWN side, the second conjunct can be used as premise in

$$\dfrac{\mathrm{M} \xrightarrow{R:^*\mathbf{cast}(m)} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\tau} [\ \mathrm{M}'\ ]}\ \textsc{Cast (T4-4)}$$

This case is proven if there exists a pair $(A_1, A_2) \in \mathcal{A}^{\smile}$ that satisfies $a \in A_1 \wedge \mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{A_1} \mathrm{T}([\ \mathrm{M}'\ ]) \wedge [\ \mathrm{M}\ ] \xrightarrow{A_2} [\ \mathrm{M}'\ ]$, and the converse of Pair 2.11 satisfies this requirement.

5: $\mathrm{T}(\mathrm{M})$ does a transition a where $\underline{a} \in \{\mathbf{t}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}$ and H does nothing.
The induction hypothesis states that

$$\exists (A_1, A_2) \in \mathcal{A}^{\smile} \,.\, a \in A_1 \wedge \mathrm{T}(\mathrm{P}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}') \wedge \mathrm{P} \xrightarrow{A_2} \mathrm{P}'$$

Define $\mathcal{A}'$ as a subset of $\mathcal{A}^{\smile}$ that contains the possible values of $(A_1, A_2) \in \mathcal{A}^{\smile}$ where A_1 are actions that $\mathrm{T}(\mathrm{P} \lll \mathrm{Q})$ can perform in mCRL2 and where A_2 are the actions that AWN should use to mimic

the actions in A_1 in order for this case to be proven:

$$\mathcal{A}' \stackrel{\text{def}}{=} \left\{ (A_1, A_2) \;\middle|\; (A_1, A_2) \in \mathcal{A}^{\smile}, \quad \mathrm{T}(\mathrm{P} \lll \mathrm{Q}) \xrightarrow{A_1} \mathrm{T}(\mathrm{P}' \lll \mathrm{Q}) \right\}$$

The following derivation is possible in mCRL2 for all $a \in A_1$ such that $(A_1, A_2) \in \mathcal{A}'$:

$$a \underline{\in} V \cup \{\tau\} \;\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{T(M)} \xrightarrow{a} \mathrm{T(M')}}\text{ Induction hypothesis}}{\mathrm{T(M)} \mid\mid \mathrm{H} \xrightarrow{a} \mathrm{T(M')} \mid\mid \mathrm{H}}\text{ Par 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\gamma_C(a)} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Comm 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{a} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Apply } \gamma_C}\text{ Rename 2}}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet a} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Apply } \{\mathbf{starcast} \to \mathbf{t}\}\bullet}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{a} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Allow 2}}{\nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{a} \nabla_V\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ T16}}{\mathrm{T}([\ \mathrm{M}\ ]) \xrightarrow{a} \mathrm{T}([\ \mathrm{M}'\ ])}$$

The derivation above is valid only for $a \in A_1$ such that $\underline{a} \subseteq V \cup \{\tau\}$. Under the constraints of this case, the derivation is also a representative derivation without alternatives (see Definition 4.9). Finally it must be observed that it is impossible for $\mathrm{T(M)}$ to produce a **newpkt** action independent of H, meaning that $\underline{a} \neq$ **newpkt**. Consequently, $\mathcal{A}'$ is as follows:

$$\{$$

$$\begin{aligned}
&(\{\ \mathbf{t}(\upsilon(D), \upsilon(R), \upsilon(m))\ \}, \{\ \tau\ \}),\\
&(\{\ \mathbf{deliver}(\upsilon(ip), \upsilon(d))\ \}, \{\ ip\!:\!\mathbf{deliver}(d)\ \}),\\
&(\{\ \mathbf{connect}(\upsilon(ip'), \upsilon(ip''))\ \}, \{\ \mathbf{connect}(ip', ip'')\ \}),\\
&(\{\ \mathbf{disconnect}(\upsilon(ip'), \upsilon(ip''))\ \}, \{\ \mathbf{disconnect}(ip', ip'')\ \})
\end{aligned}$$

$$\mid\quad m : \mathrm{MSG};\quad ip, ip', ip'' : \mathrm{IP};\quad d : \mathrm{DATA};\quad dests, R, D : \mathrm{Set(IP)};\quad R \subseteq D\quad \}$$

For all $(A_1, A_2) \in \mathcal{A}'$ the actions A_1 in mCRL2 can be mimicked by the actions A_2 in AWN by means of the inference rules

$$\frac{\mathrm{M} \xrightarrow{\tau} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\tau} [\ \mathrm{M}'\ ]}\ \textsc{Internal}\ (\text{T4-3})$$

$$\frac{\mathrm{M} \xrightarrow{ip:\mathbf{deliver}(d)} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{ip:\mathbf{deliver}(d)} [\ \mathrm{M}'\ ]}\ \textsc{Deliver}\ (\text{T4-3})$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{connect}(ip,ip')} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\mathbf{connect}(ip,ip')} [\ \mathrm{M}'\ ]}\ \textsc{Connect}\ (\text{T4-2})$$

$$\frac{\mathrm{M} \xrightarrow{\mathbf{disconnect}(ip,ip')} \mathrm{M}'}{[\ \mathrm{M}\ ] \xrightarrow{\mathbf{disconnect}(ip,ip')} [\ \mathrm{M}'\ ]}\ \textsc{Disconnect}\ (\text{T4-2})$$

respectively. Therefore the induction hypothesis holds for this case.

6: $\mathrm{T(M)}$ does a transition a where $\underline{a} \notin \{\mathbf{t}, \mathbf{starcast}, \mathbf{newpkt}, \mathbf{deliver}, \mathbf{connect}, \mathbf{disconnect}\}$ and H does nothing. This situation fails to generate behavior for $\mathrm{T}([\ \mathrm{M}\ ])$ because no derivation similar to the one in the Lemma 4.6-proof for Newpkt (T4) is possible:

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{T(M)} \xrightarrow{a} \mathrm{T(M')}}{\mathrm{T(M)} \mid\mid \mathrm{H} \xrightarrow{a} \mathrm{T(M')} \mid\mid \mathrm{H}}\text{ Par 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\gamma_C(a)} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Comm 2}}{\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{a} \Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Apply } \gamma_C}{\rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M)} \mid\mid \mathrm{H}) \xrightarrow{\{\mathbf{starcast}\to\mathbf{t}\}\bullet a} \rho_{\{\mathbf{starcast}\to\mathbf{t}\}}\Gamma_C(\mathrm{T(M')} \mid\mid \mathrm{H})}\text{ Rename 2}$$

The Allow 2 operator cannot be applied next because $\underline{a} \notin V \cup \{\tau\}$. Since this means that $\mathrm{T}([\ \mathrm{M}\ ])$ cannot do a transition in mCRL2 under the circumstances specified in this particular case, there is no behavior for AWN to mimic.