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Ad hoc Routing in Mesh Networks using Algebra

Peter Höfner

Australian Government
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Some Background (Recap)

- Routing protocols
  - find a route (in a dynamic topology)
  - properties
    - route correctness (if a route is found, the route is actually present)
    - route discovery (if a route exist, the route is found)
    - loop freedom (packets do not circulate)
    - packets are delivered (eventually)

- Routing tables
  - collect (known) data
    - IP address, local connections, next hops ...

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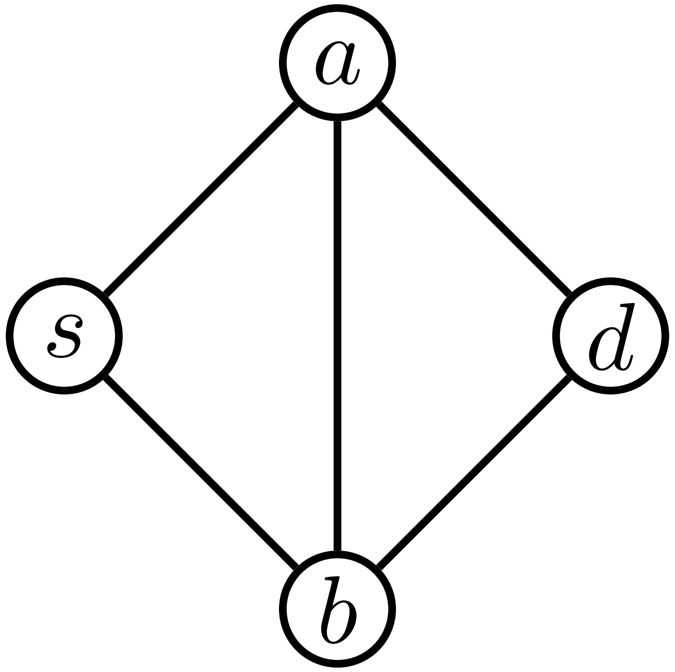
Some Background (Recap)

- Goal: Study routing algorithms algebraically
  – inspired by the standard, popular routing protocol AODV
- Ad hoc on-demand distance vector protocol (AODV)
  – main Mechanism
    - if route is needed
      BROADCAST RREQ
    - if node has information about a destination
      UNICAST RREP
    - if unicast fails or link break is detected
      GROUPCAST RERR
  – routing table
    - destination address
    - next hop (not the entire path)
    - length of the route
    - parameter about freshness
      (sequence numbers)



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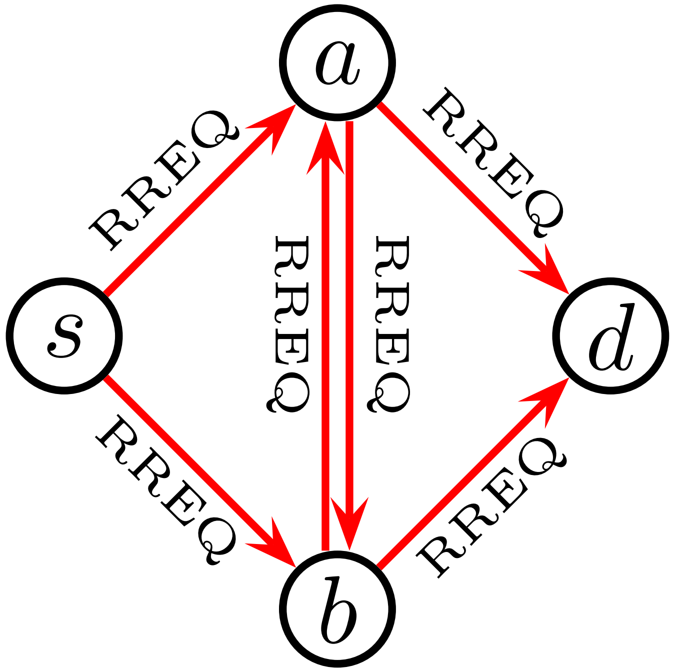

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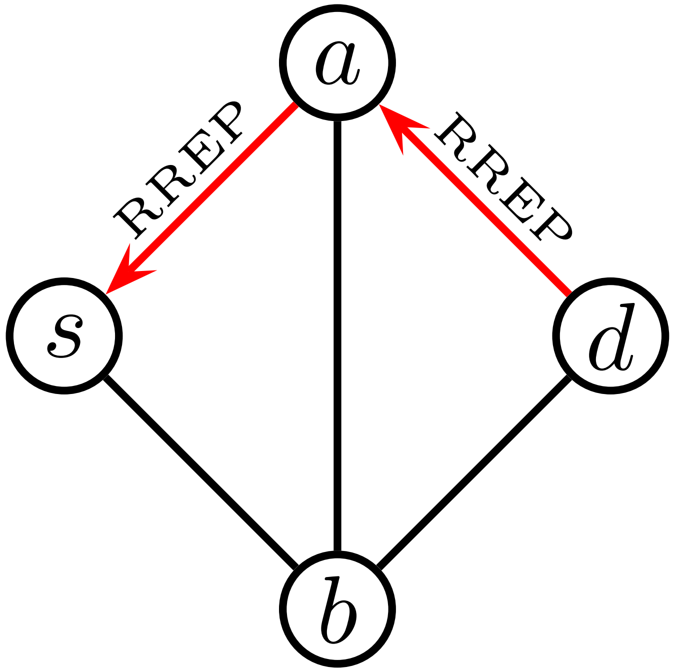

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Towards an Algebra

- Algebra
  - offer operations for main primitives (broadcast, unicast, ...)
  - model properties such as loop freedom algebraically
- Operators
  - choice
    - if a node has the choice between two routes, it has to choose one
  - composition
    - if two routes are known they can be combined

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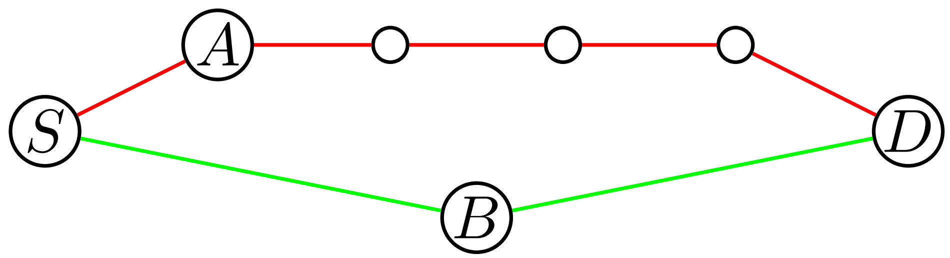
Algebraic Operations (minimal requirements)

- Routing table entries (no sequence number so far)

$$(\texttt{nhip}\,,\,\texttt{hops})$$

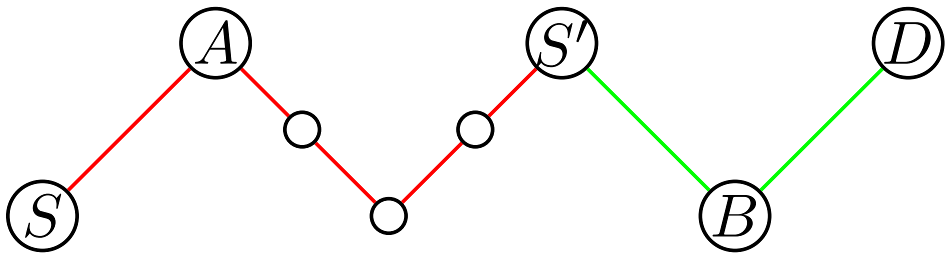
- Choice (lexicographical order):

$$(A,5)+(B,2)=(B,2)$$



- Multiplication (destination and source must coincide)

$$(A,5)\cdot(B,2)=(A,7)$$



- Special symbols: $(\_,0)$, $(\_,\infty)$

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Underlying Structure

- Both (+) and (·) structures form monoids
- Multiplication distributes over addition
- Lifts to matrices
- Use semirings and Kleene algebras to study routing protocols?
- inspired by Backhouse, Carré, Griffin, Sobrinho

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Routing Algebra – Elements, Operators

- Matrices over routing table entries

$$\begin{array}{cc}
 & \begin{array}{ccccc} A & B & C & D & \cdots \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ \vdots \end{array} &
\begin{pmatrix}
(_, 0) & (B, 1) & (B, 2) & (_, \infty) & \\
(A, 1) & (_, 0) & (C, 1) & (_, \infty) & \cdots \\
(_, \infty) & (B, 1) & (_, 0) & (_, \infty) & \\
(_, \infty) & (_, \infty) & (_, \infty) & (_, 0) & \\
 & \vdots & & & \ddots
\end{pmatrix}
\end{array}$$

routing table of A

“routes” to B

- standard matrix operations
- further abstraction possible (semirings, test, domain, modules ...)

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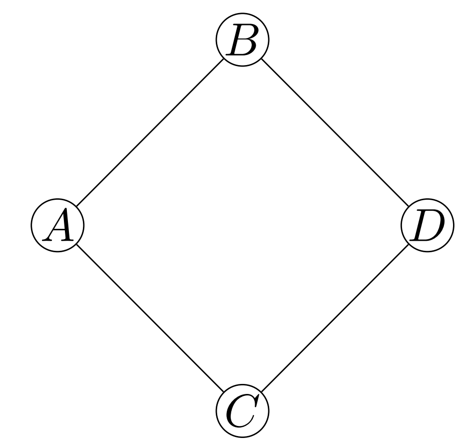
Example

- A route request is broadcast



$$\begin{pmatrix} (\_,0) & (B,1) & (C,1) & (\_,\infty) \\ (A,1) & (\_,0) & (\_,\infty) & (D,1) \\ (A,1) & (\_,\infty) & (\_,0) & (D,1) \\ (\_,\infty) & (B,1) & (C,1) & (\_,0) \end{pmatrix} \bullet \begin{pmatrix} (\_,0) & (\_,\infty) & (\_,\infty) & (\_,\infty) \\ (\_,\infty) & (\_,\infty) & (\_,\infty) & (\_,\infty) \\ (\_,\infty) & (\_,\infty) & (\_,\infty) & (\_,\infty) \\ (\_,\infty) & (\_,\infty) & (\_,\infty) & (\_,\infty) \end{pmatrix} \bullet \begin{pmatrix} (\_,0) & (B,1) & (\_,\infty) & (\_,\infty) \\ (\mathbf{D},\mathbf{3}) & (\_,0) & (\_,\infty) & (\_,\infty) \\ (A,1) & (\_,\infty) & (\_,0) & (D,1) \\ (C,2) & (\_,\infty) & (C,1) & (\_,0) \end{pmatrix}$$

topology   sender   routing table

$$= \begin{pmatrix} (\_,0) & (B,1) & (\_,\infty) & (\_,\infty) \\ (\mathbf{A},\mathbf{1}) & (\_,0) & (\_,\infty) & (\_,\infty) \\ (A,1) & (\_,\infty) & (\_,0) & (D,1) \\ (C,2) & (\_,\infty) & (C,1) & (\_,0) \end{pmatrix}$$

updated routing table

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Sending of Messages

- Sending messages

$$a + p \cdot b \cdot q \cdot (1 + c)$$

with

- a known knowledge (snapshot)
- p, q sender and receiver
- b topology
- $p \cdot b \cdot q$ restricted topology
- $1 + c$ possible updates/information sent

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Calculating with Messages

Definition: messages can be defined as

$$\mathtt{msg}(a, b, c) = a + b \cdot (1 + c) \qquad (1 \leq b)$$

Properties:

- If the c and c' is fixed (does not change when sending a message), the order of sending does not matter, i.e.,

$$\mathtt{msg}(\mathtt{msg}(a, b, c), b', c') = \mathtt{msg}(\mathtt{msg}(a, b', c'), b, c) \ .$$

- If different messages are sent via a shared topology b, the messages can be sent in parallel, i.e.,

$$\mathtt{msg}(\mathtt{msg}(a, b, c), b, c') = \mathtt{msg}(a, b, c + c') \ .$$

- If the same message is sent via different connections, connections can be joined, i.e.,

$$\mathtt{msg}(\mathtt{msg}(a, b, c), b', c) = \mathtt{msg}(a, b + b', c) \ .$$

These properties as well as others can be automatically proven (e.g. by Prover9)

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Distributing a message (Flooding the Network)

- Forwarding message

$$\begin{aligned} & \mathtt{msg}(\mathtt{msg}(a,b,c),b',b\cdot c) \\ = \quad & a+b+b\cdot c+b'+b'\cdot b\cdot c \\ \leq \quad & a+b'+b'\cdot b+b'\cdot b\cdot c \\ = \quad & a+b'(1+b+b\cdot c) \\ = \quad & \mathtt{msg}(a,b',b+b\cdot c) \end{aligned}$$

- knowledge after forwarding a message once can be approximated by sending a single message via b′ with knowledge of the first topology b and the learnt component $b \cdot c$
- in case the topology does not change

$$\mathtt{msg}(\mathtt{msg}(a,b,c),b,b\cdot c) = \mathtt{msg}(a,b,b\cdot c)\ .$$

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Distributing a message (Flooding the Network)

- Broadcasting a message

$$\begin{aligned}\mathtt{msg}(a, b, b^* \cdot c) &= a + b \cdot (1 + b^* \cdot c) \\ &= a + b + b \cdot c + b \cdot b \cdot c + b \cdot b \cdot b \cdot c + \ldots\ .\end{aligned}$$

where * is Kleene star

- Single source

$$\mathtt{msg}(a, b \cdot |b^*\rangle p, b^* \cdot p) = a + b \cdot |b^*\rangle p + b^* \cdot p$$

with sender p $(p \leq 1, \mathrm{test})$
and receivers $|b^i\rangle p$

$$|a\rangle p \leq q \Leftrightarrow \neg q \cdot a \cdot p \leq 0 \ \text{ and } \ |a \cdot b\rangle p = |a\rangle(|b\rangle p)$$

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Unicast and Broadcast

- By varying the topology one can model broadcast, multicast and unicast.
- Modal operators can be used to characterise stopping criteria (of AODV)
  (use $b \cdot |a]\neg q$ as topology, where $|a]p = \neg|a\rangle\neg p$)

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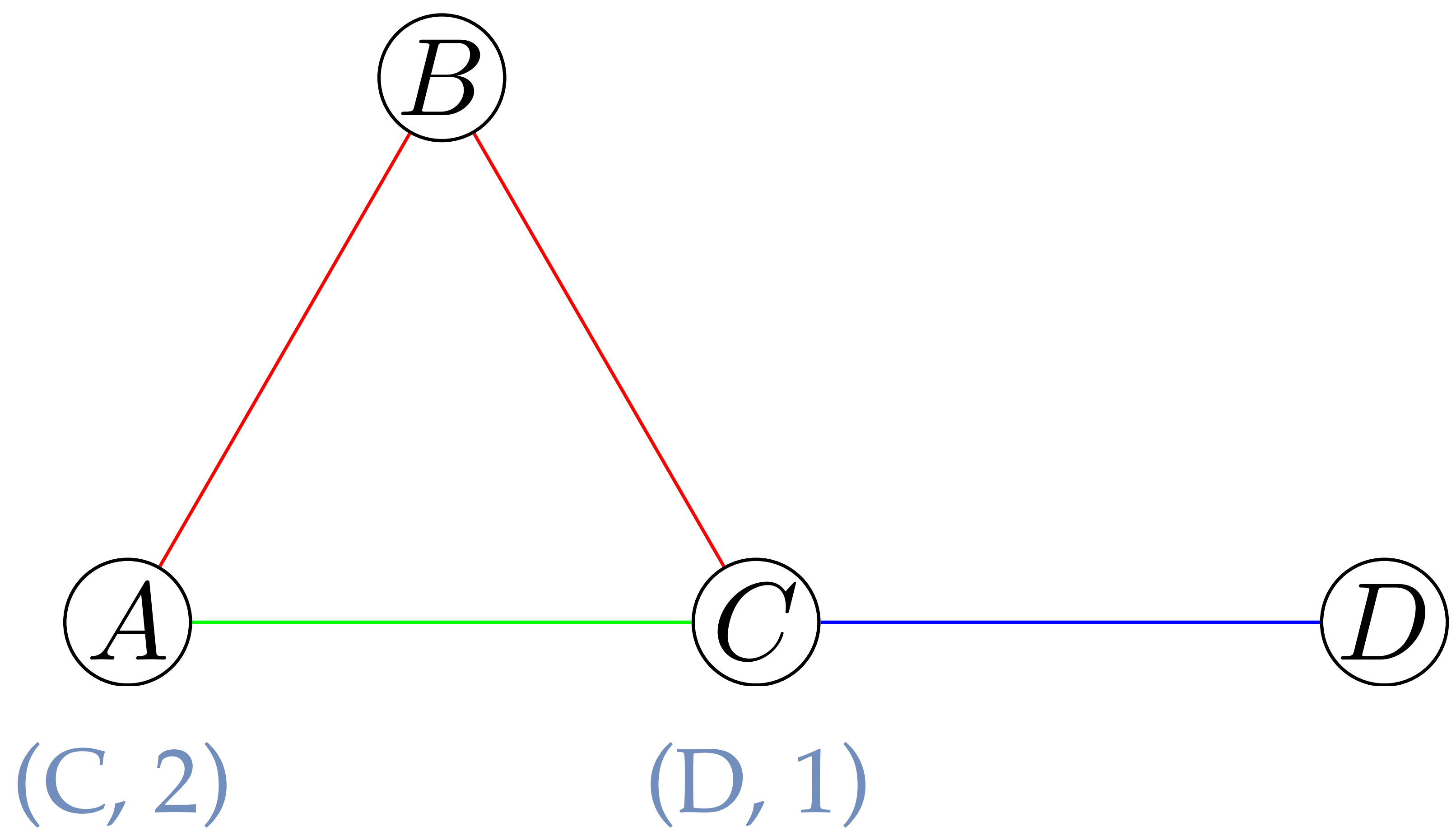
Loop Freedom and Wrong Routing Protocols

- Routing protocols (on top of dynamic topologies) must avoid routing loops
  - C and A have established routes to D



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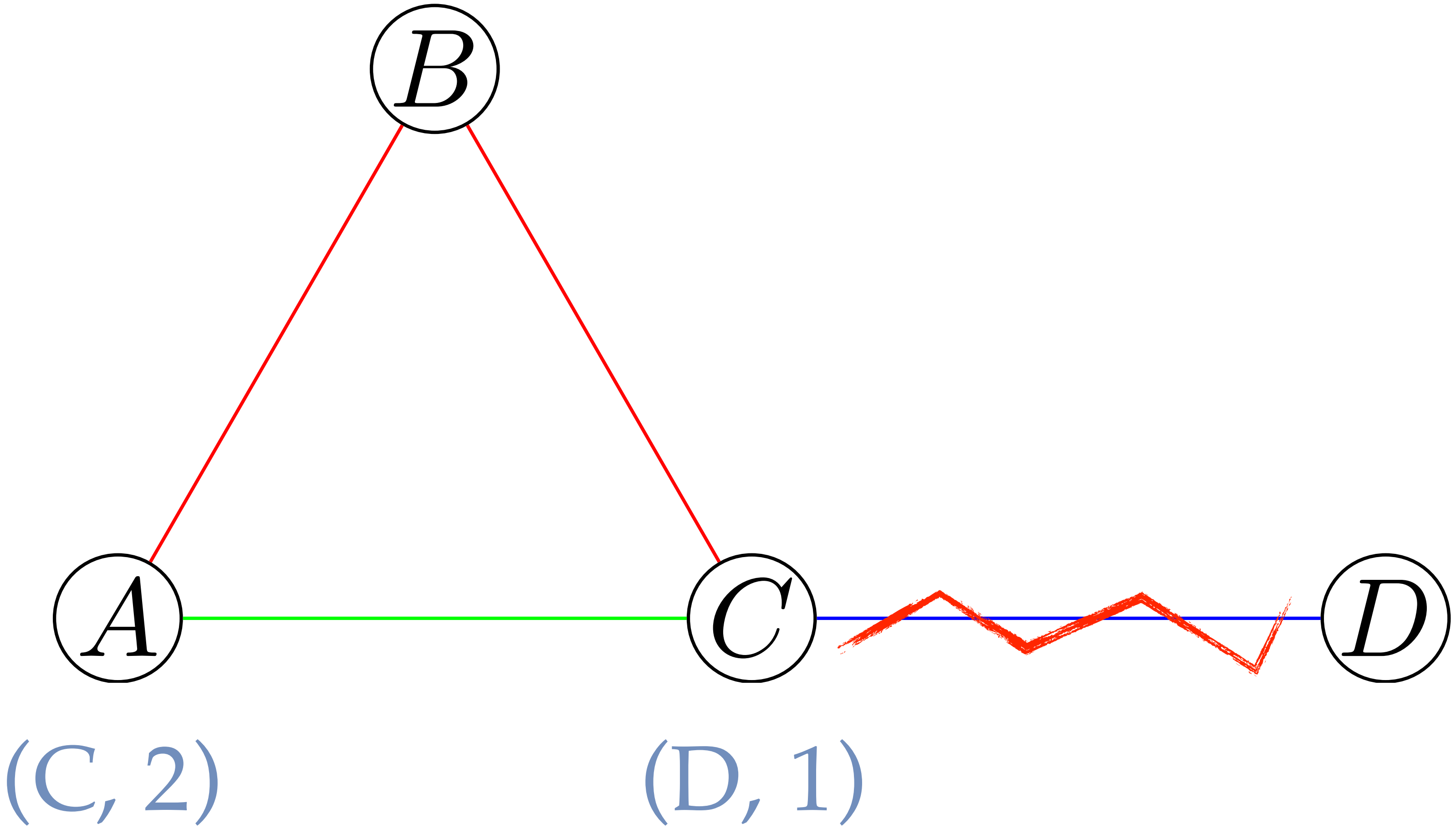

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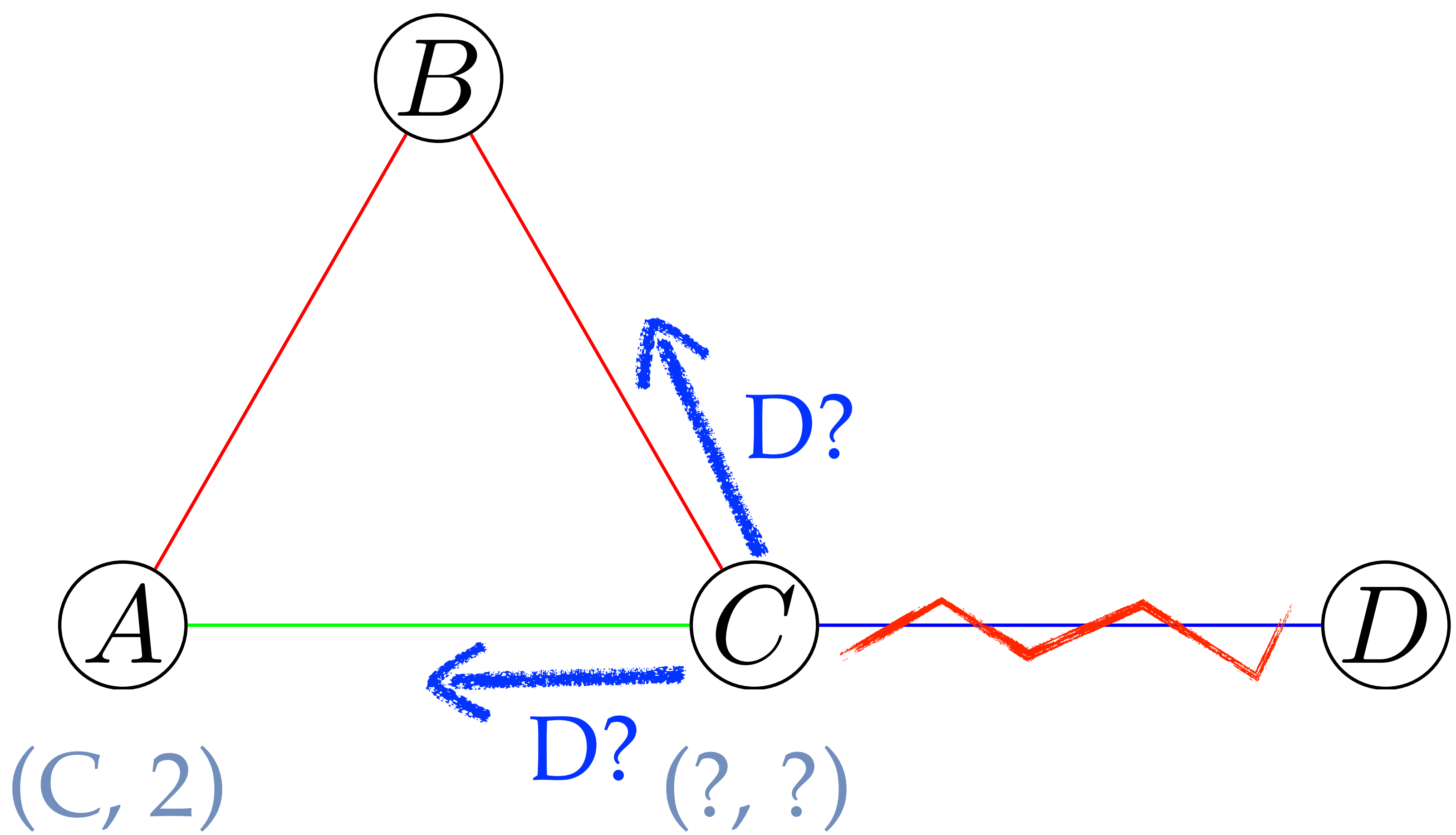
Loop Freedom and Wrong Routing Protocols

- Routing protocols (on top of dynamic topologies) must avoid routing loops
  - C send request to find route to D



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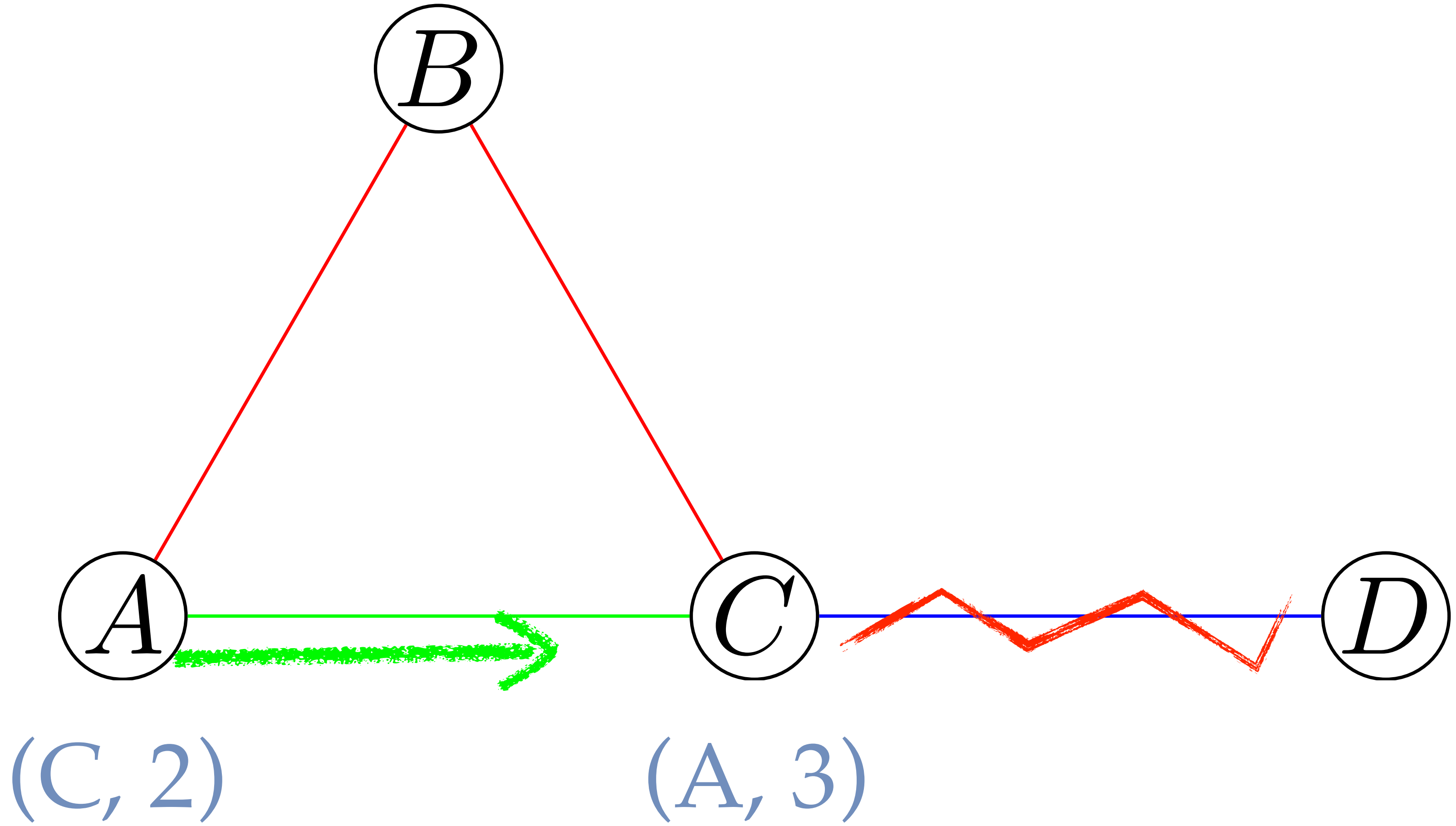
Loop Freedom and Wrong Routing Protocols

- Routing protocols (on top of dynamic topologies) must avoid routing loops
  - A answers with a route reply



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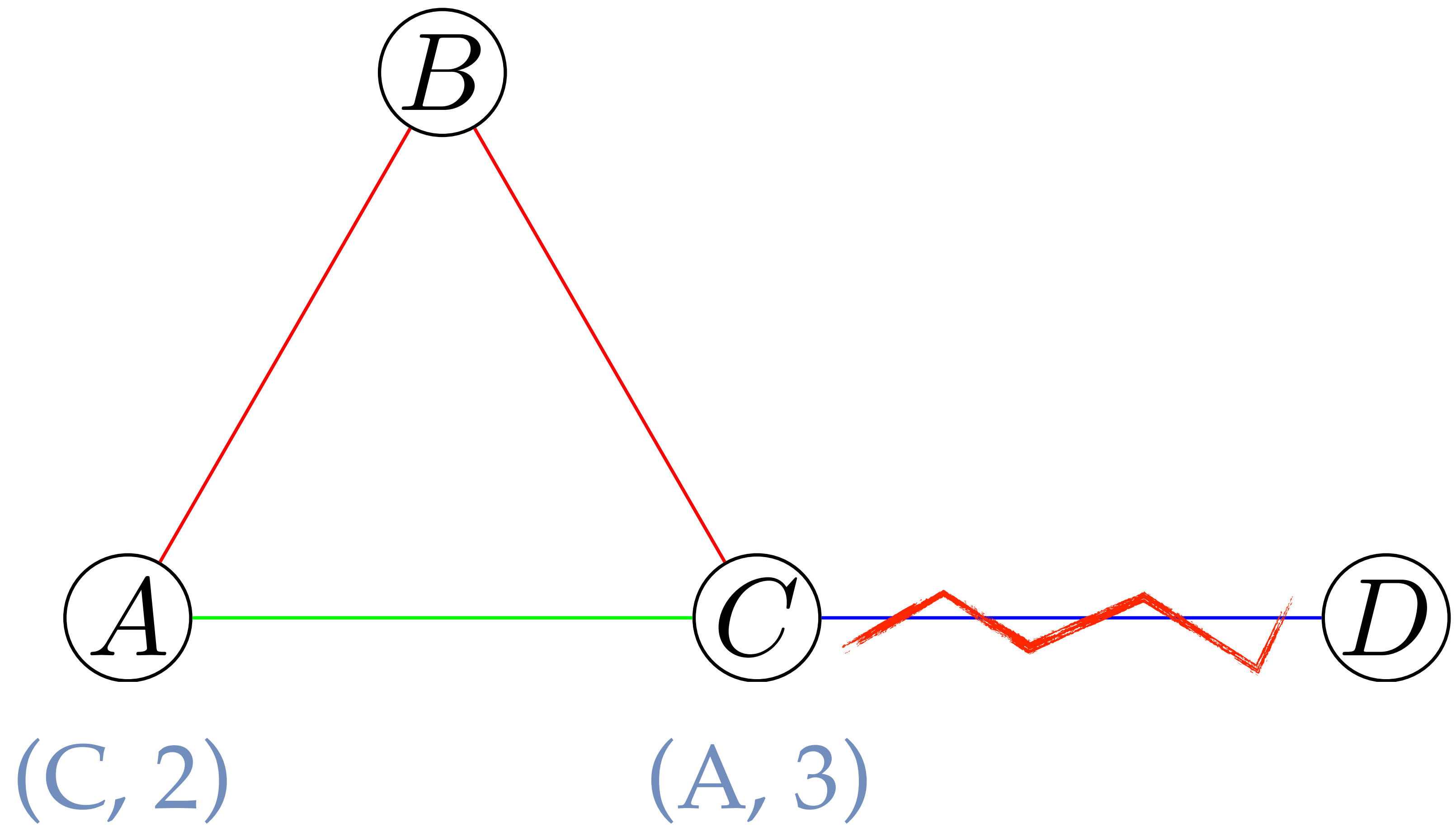
Loop Freedom and Wrong Routing Protocols

- Routing protocols (on top of dynamic topologies) must avoid routing loops
  - A routing loop has been established



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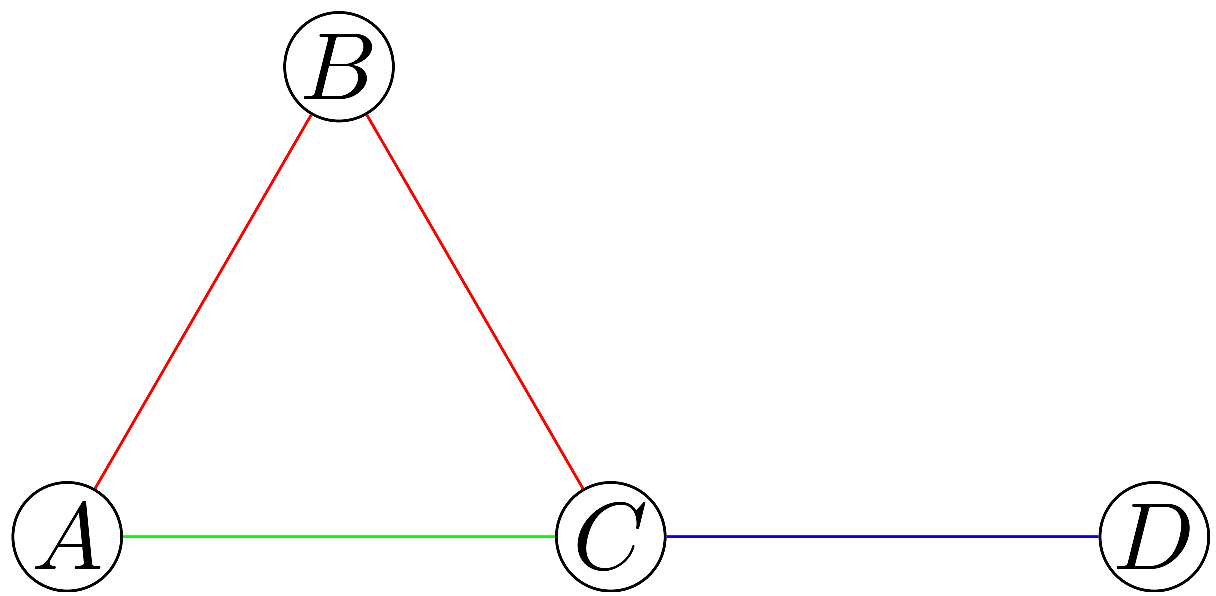
Guaranteeing Loop Freedom

- add an attribute “freshness”
  - routing information records the “destination sequence number”, i.e. the sequence number reported by messages “coming from” that destination: (nhop, hops, dsn)



$$
\begin{aligned}
r \cdot b &= (B, 2, 5) \cdot (D, 1, 10) &= (B \cdot D, 2+1, \max(5, 10)) &= (B, 3, 10) \\
g \cdot b &= (C, 1, 3) \cdot (D, 1, 10) &= (C \cdot D, 1+1, \max(3, 10)) &= (C, 2, 10)
\end{aligned}
$$

$$r \cdot b + g \cdot b \quad \neq \quad (r+g) \cdot b$$

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Towards a Solution

- The problem is that in our algebraic setting the topology would carry sequence numbers.
  - intuitively this does not make sense

- Idea: distinguish between routing tables and topologies
  - routing table
    - knowledge of nodes
    - information sent via the topology
  - topology
    - information about (current) connectivity

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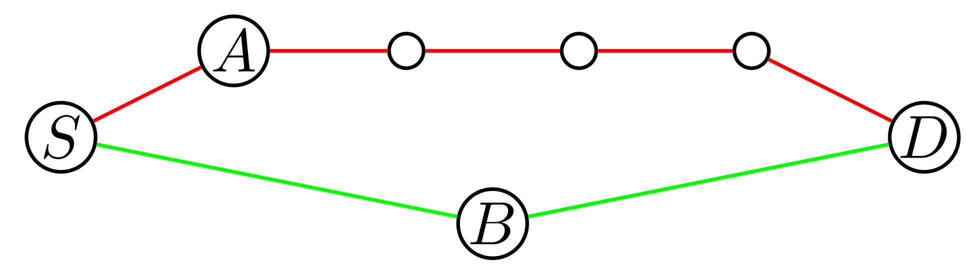
Algebraic Operations (incl. sequence numbers)

- Topologies (no sequence number)

$$(\texttt{nhip}\,,\,\texttt{hops})$$

- Choice (lexicographical order):

$$(A,5)+(B,2)=(B,2)$$



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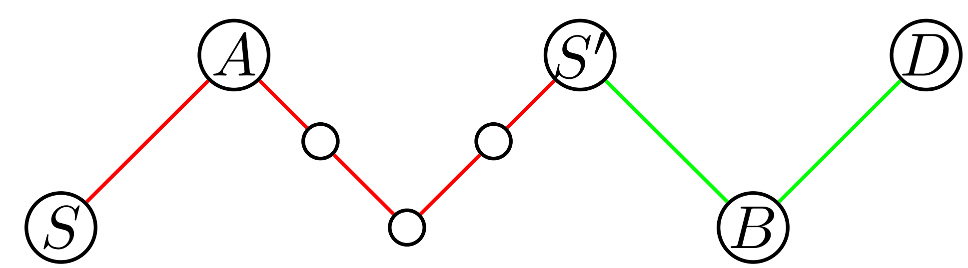

- Special symbols: $(\_,0)$, $(\_,\infty)$

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Algebraic Operations (incl. sequence numbers)

- Routing table entries

$$(\texttt{nhip}\,,\,\texttt{hops}\,,\,\texttt{sqn})$$

- Choice (on topologies):

$$(A,5,10)\sqcup(B,2,3)=(A,5,10)$$

- Multiplication does not exist
- Special symbol: $(\_,\infty,\infty)$

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Algebraic Operations (incl. sequence numbers)

- Mapping topologies to routing tables (updating routing tables)

$$(A, 5) : (B, 2, 5) = (A, 7, 5)$$

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Underlying Structure

- (+), (·), (⊔) structures form monoids
- multiplication distributes over addition
- scalar product (:) satisfies

$$\begin{array}{lrcl}
\textit{unit} & 1 : r & = & r \ , \\
\textit{distributivity} & (t + t') : r & = & (t : r) \sqcup (t' : r) \\
\textit{distributivity} & t : (r \sqcup r') & = & (t : r) \sqcup (t : r') \\
\textit{associativity} & (t \cdot t') : r & = & t : (t' : r)
\end{array}$$

- together this structure forms a Kleene Module
  – (à la Leiß)
- lift to matrices

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From Kleene Algebra to Kleene Modules

- all theory presented can be transferred to Kleene modules
  - e.g. sending messages

$$\mathtt{msg}(a, b, c) = a \sqcup b : c$$

$$\begin{aligned}
\mathtt{msg}(a, b, b^* : c) &= a \sqcup b \cdot (1 + b^* : c) \\
\mathtt{msg}(a, b \cdot |b^*\rangle p, b^* : p') &= a \sqcup b \cdot |b^*\rangle p : \mathsf{e} \sqcup b^* : p' \\
&= a \sqcup b \cdot \ulcorner(b^*) : p' \sqcup b^* : p'
\end{aligned}$$

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On-Going Work

- Modelling AODV
  - require additional operations for
    - incrementing sequence numbers
    - invalidating routes ...
- Unicast
  - so far unicast was modelled by a given topology
  - can this topology determined automatically?
  - maybe via fixpoints

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On-Going Work

- Properties of Routing Protocol
  - route correctness (by construction)
  - route discovery

$$s \cdot P \cdot d \neq 0$$

  - route optimality (for static topology b)

$$s \cdot P \cdot d = s \cdot b^* \cdot d$$

  - loop freedom
    - details still open
    - use “inverse” of scalar product to forget sequence numbers
    - then compare with identity

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Future Work

- Formalise main aspects of AODV
  - AODV works on 4-tuples rather than triples (fits well in the theory of modules)
- Try to derive a “correct” protocol from algebraic specification
- Maybe introduce time in the model
  - (seminal work by Hoare, von Karger, Hayes)

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- Title: **Ad hoc Routing in Mesh Networks using Algebra**
  - Author: Peter Höfner
  - Affiliation: NICTA (National ICT Australia) and UNSW
  - Research Interest: Modelling and Verification of (Software Systems) using formal methods such as algebraic structures. At the moment focus on routing and communication protocols
  - Abstract: At the meeting in Rome I gave an overview of formal modelling and analysis of routing protocols for wireless mesh networks (WMNs). Afterwards I was asked to present details about the methods used. This talk presents some more details about the algebra used to model main aspect of routing protocols.
  - time: 30-40 min

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