Towards an Algebra of Routing Tables

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What is the Problem?

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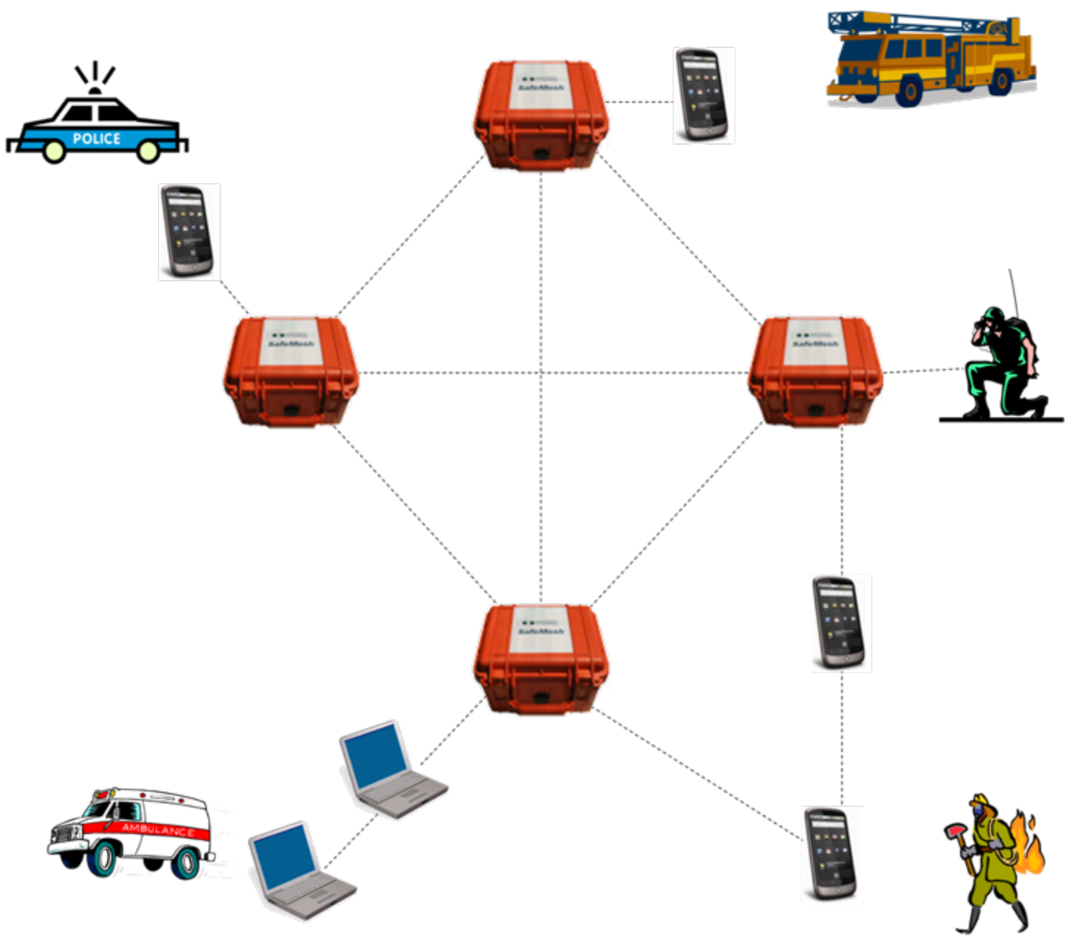
- Wireless Mesh Networks
  - key advantage: no backhaul wiring required
  - quick and low cost deployment
- Applications
  - public safety (e.g. CCTV)
  - emergencies (e.g. earthquakes)
  - mobile phone services
  - transportation
  - mining
  - military actions/counter terrorism
  - ...



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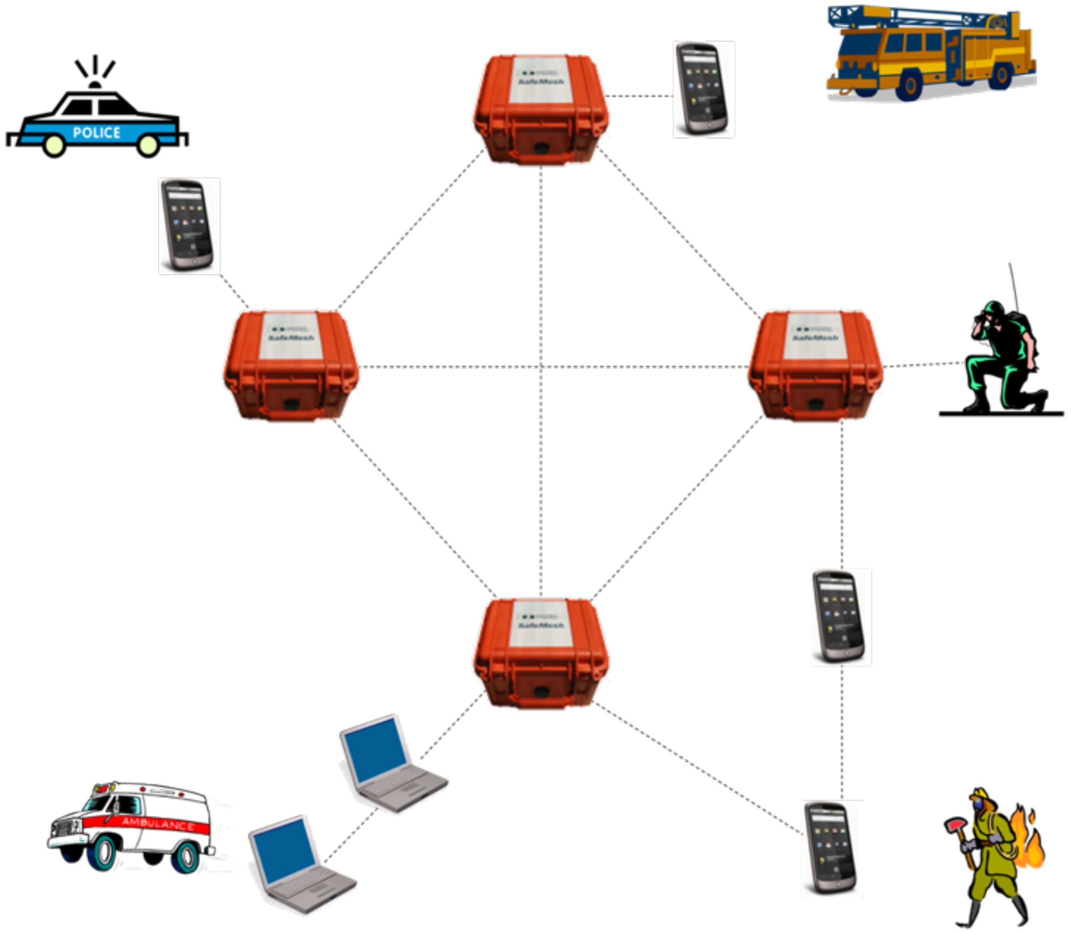
What is the Problem?

- WMNs promise to be fully
  – self-configuring
  – self-healing
  – self-optimising



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What is the Problem?

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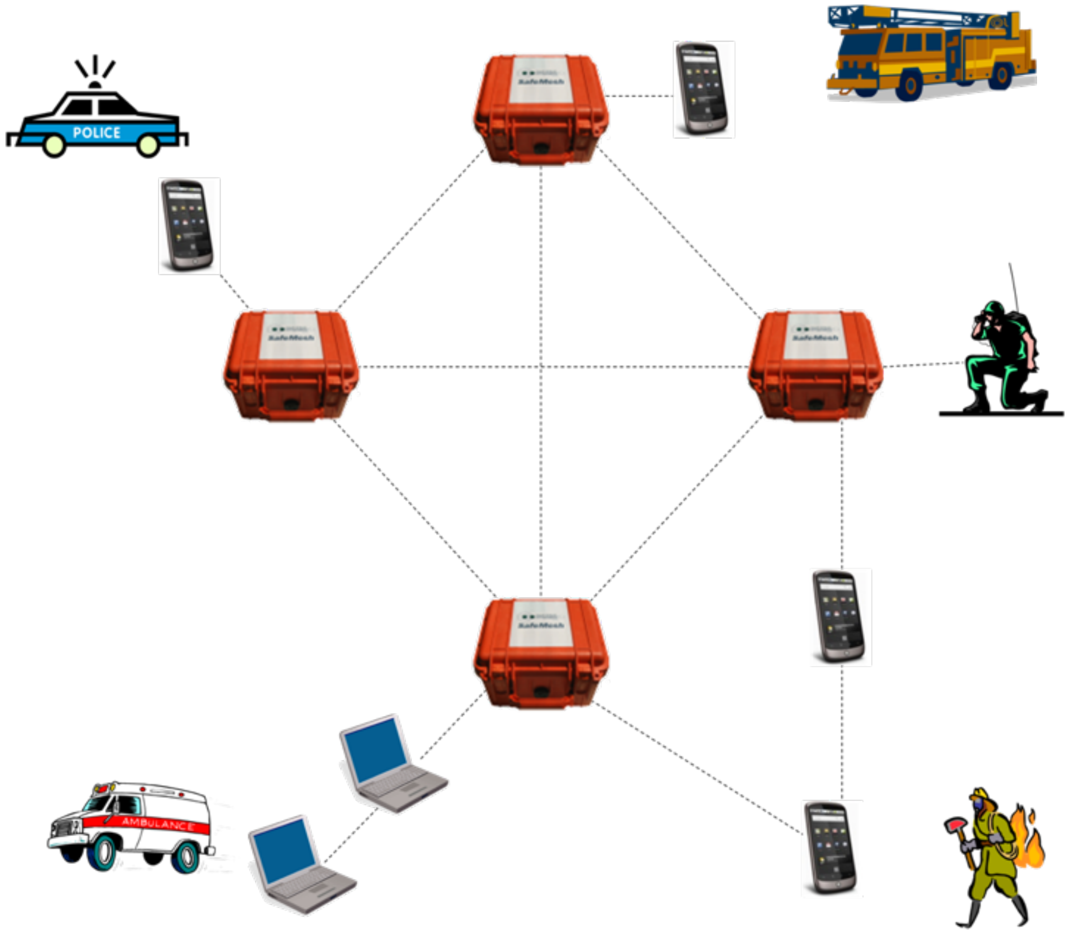
- WMNs promise to be fully
  - self-configuring
  - self-healing
  - self-optimising
- **THAT IS NOT TRUE** (in reality)
- Limitations in reliability and performance
- Limitations confirmed by
  - end users (e.g. police)
  - own experiments
    - Cisco, Motorola, Firetide, ...
  - industry



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Formal Methods for Mesh Networks

- Goal
  - model, analyse, verify and increase the performance of wireless mesh protocols
  - develop suitable formal methods techniques
- Benefits
  - more reliable protocols
  - finding and fixing bugs
  - better performance
  - proving correctness
  - reduce “time-to-market”

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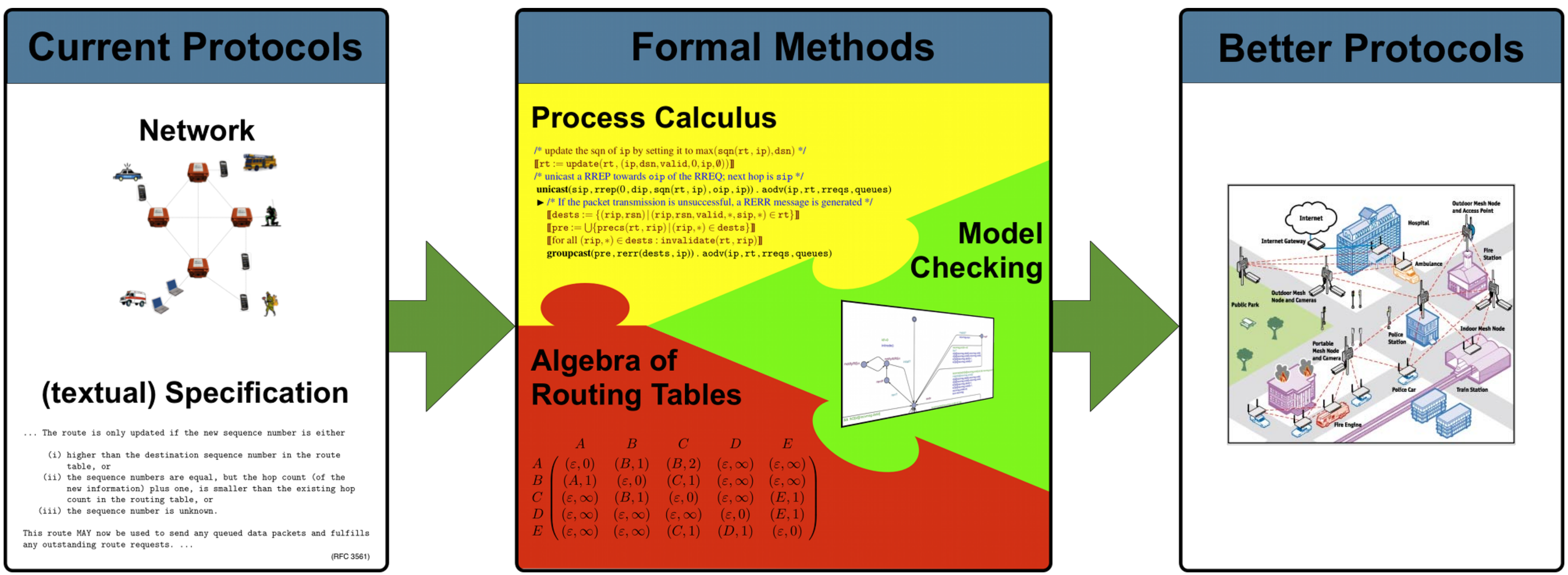
Formal Methods for Mesh Networks

- Main Methods used so far
  - process algebra
  - model checking
  - **algebra of routing tables**



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Ad Hoc On-Demand Distance Vector Protocol

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- Routing protocol for WMNs

- Ad hoc (network is not static)
- On-Demand (routes are established when needed)
- Distance (metric is hop count)
- Vector (routing table has the form of a vector)

- Developed 1997-2001 by Perkins, Beldig-Royer and Das (University of Cincinnati)

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Ad Hoc On-Demand Distance Vector Protocol

- AODV control messages
  - route request (RREQ)
  - route reply (RREP)
  - route error message (RERR)
  - (Hello messages)

- Information at nodes
  - own IP address
  - a local sequence number (freshness/timer)
  - a routing table
    - local knowledge
    - entries: $(\texttt{dip}\,,\,\texttt{dsn}\,,\,\texttt{val}\,,\,\texttt{hops}\,,\,\texttt{nhip}\,,\,\texttt{pre})$

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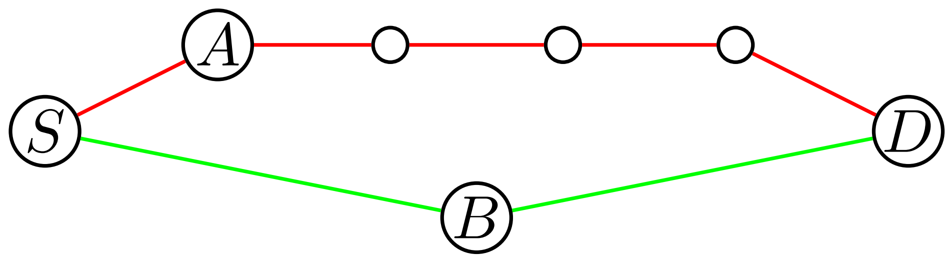
Routing Algebra – Elements, Operators

- Routing table entries (no sequence number so far)

$$(\texttt{nhip}\,,\,\texttt{hops})$$

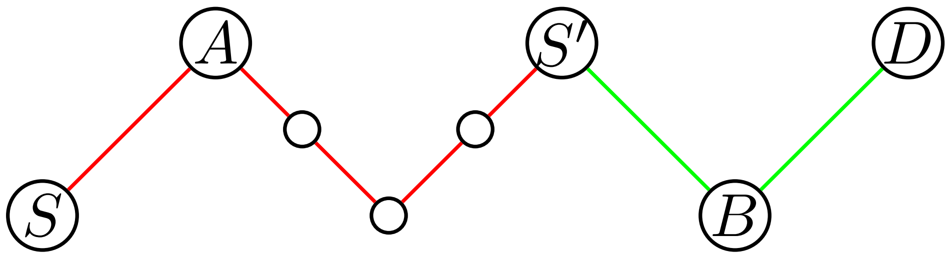
- Choice:

$$(A,5)+(B,2)=(B,2)$$



- Multiplication (destination and source must coincide)

$$(A,5)\cdot(B,2)=(A,7)$$



- Special symbols: $(\_,0)$, $(\_,\infty)$
- Idea: back to Backhouse, Carré, Griffin, Sobrinho

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Routing Algebra – Elements, Operators

- Matrices over routing table entries

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ \vdots \end{array}
\begin{array}{ccccc} A & B & C & D & \dots \end{array}
$$

$$
\begin{array}{c} A \\ B \\ C \\ D \\ \vdots \end{array}
\begin{pmatrix}
(\_, 0) & (B, 1) & (B, 2) & (\_, \infty) & \\
(A, 1) & (\_, 0) & (C, 1) & (\_, \infty) & \dots \\
(\_, \infty) & (B, 1) & (\_, 0) & (\_, \infty) & \\
(\_, \infty) & (\_, \infty) & (\_, \infty) & (\_, 0) & \\
 & \vdots & & & \ddots
\end{pmatrix}
\text{routing table of } A
$$

“routes” to B

- standard matrix operations
- further abstraction possible
  (semirings, test, domain, modules ...)

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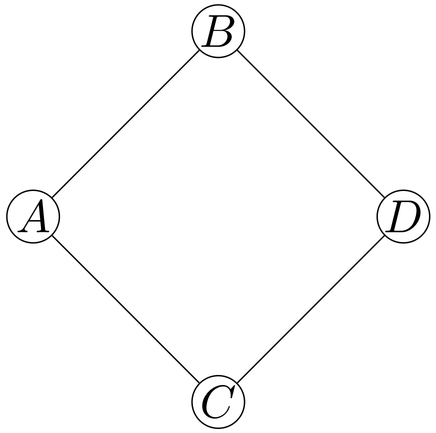
Example

- A route request is broadcast



$$
\begin{pmatrix}
(\_,0) & (B,1) & (C,1) & (\_,\infty) \\
(A,1) & (\_,0) & (\_,\infty) & (D,1) \\
(A,1) & (\_,\infty) & (\_,0) & (D,1) \\
(\_,\infty) & (B,1) & (C,1) & (\_,0)
\end{pmatrix}
\bullet
\begin{pmatrix}
(\_,0) & (\_,\infty) & (\_,\infty) & (\_,\infty) \\
(\_,\infty) & (\_,\infty) & (\_,\infty) & (\_,\infty) \\
(\_,\infty) & (\_,\infty) & (\_,\infty) & (\_,\infty) \\
(\_,\infty) & (\_,\infty) & (\_,\infty) & (\_,\infty)
\end{pmatrix}
\bullet
\begin{pmatrix}
(\_,0) & (B,1) & (\_,\infty) & (\_,\infty) \\
(\mathbf{D},\mathbf{3}) & (\_,0) & (\_,\infty) & (\_,\infty) \\
(A,1) & (\_,\infty) & (\_,0) & (D,1) \\
(C,2) & (\_,\infty) & (C,1) & (\_,0)
\end{pmatrix}
$$

topology   sender   routing table

$$
= \begin{pmatrix}
(\_,0) & (B,1) & (\_,\infty) & (\_,\infty) \\
(\mathbf{A},\mathbf{1}) & (\_,0) & (\_,\infty) & (\_,\infty) \\
(A,1) & (\_,\infty) & (\_,0) & (D,1) \\
(C,2) & (\_,\infty) & (C,1) & (\_,0)
\end{pmatrix}
$$

updated routing table

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Further Abstraction

- Interpret matrix as an arbitrary element of a semiring
- Kleene algebra allows iteration,
- (Co)Domain and tests model projections

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Sent Messages

- sending messages

$$a + p \cdot b \cdot q \cdot (1 + c)$$

- by distributivity

$$a \;\;+\;\; p \cdot b \cdot q \;\;+\;\; p \cdot b \cdot q \cdot c$$

  snapshot, 1-hop connection learnt, content sent
- broadcast, unicast, groupcast are the same (modelled by different topologies)
- Kleene star models flooding the network (modal operators terminate flooding)

- QUESTION: Can unicast modelled purely algebraically?

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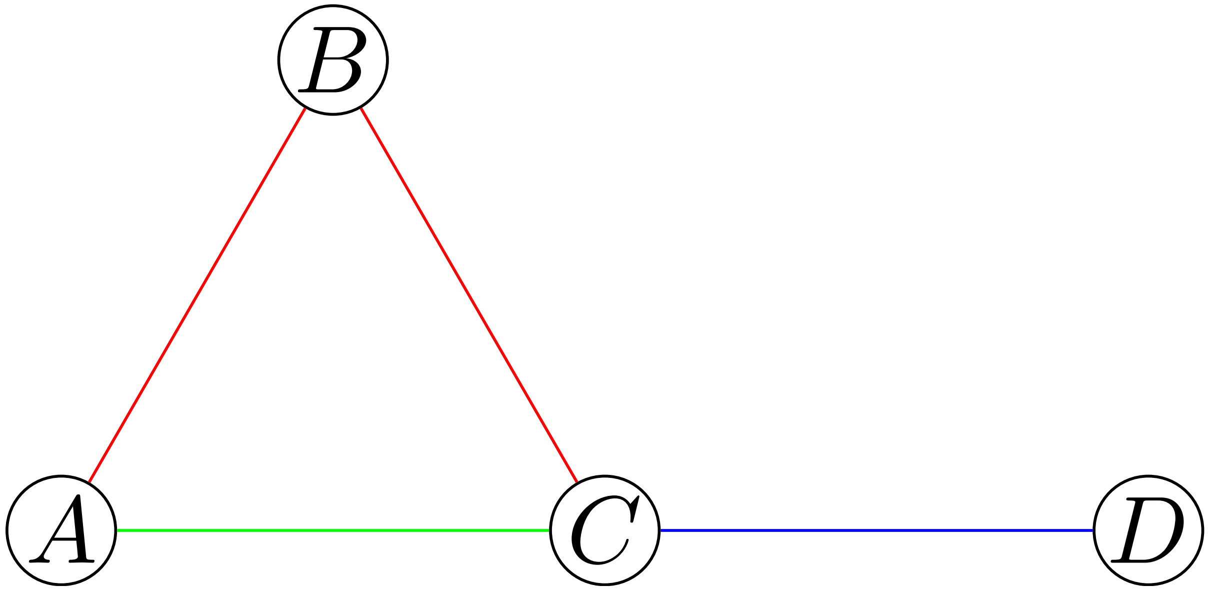
Lost and Found

- Adding sequence numbers



$$
\begin{aligned}
r \cdot b &= (B,2,5)\cdot(D,1,10) &= (B\cdot D, 2+1, \max(5,10)) &= (B,3,10)\\
g \cdot b &= (C,1,3)\cdot(D,1,10) &= (C\cdot D, 1+1, \max(3,10)) &= (C,2,10)
\end{aligned}
$$

$$r\cdot b + g\cdot b \neq (r+g)\cdot b$$

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Lost and Found

- Restrict multiplication
  - partial defined operation
  - only topologies allowed on the left-hand side
  - Kleene star has to be adapted

- Module like structure
  (scalars are subalgebra)

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Miscellaneous

- Ad hoc prototype in Haskell
- Theorems at algebraic level proven with Prover9
- Use Isabelle/HOL to switch between model and algebra

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Future Work

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- Include sequence numbers
- Important properties loop freedom, route correctness
- Improvement/refinement
- So far concentrated on AODV
  - well known, IETF standard, known limitations
- Extend formal methods to other protocols
  - OSLR, DYMO, DLR ...
- Add further necessary concepts
  - time, probability

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From **imagination** to **impact**

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From **imagination** to **impact**

AODV – An Example

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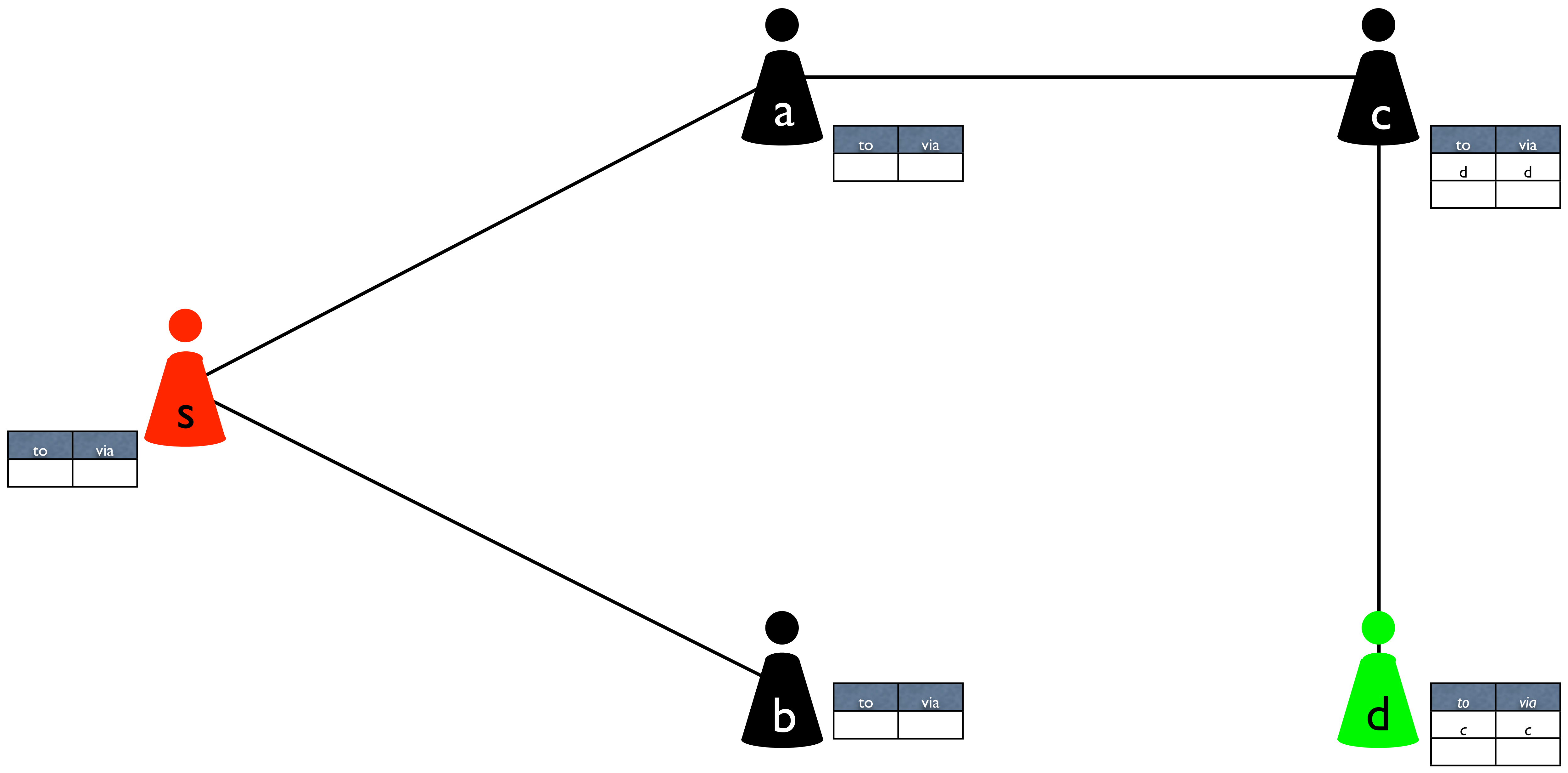
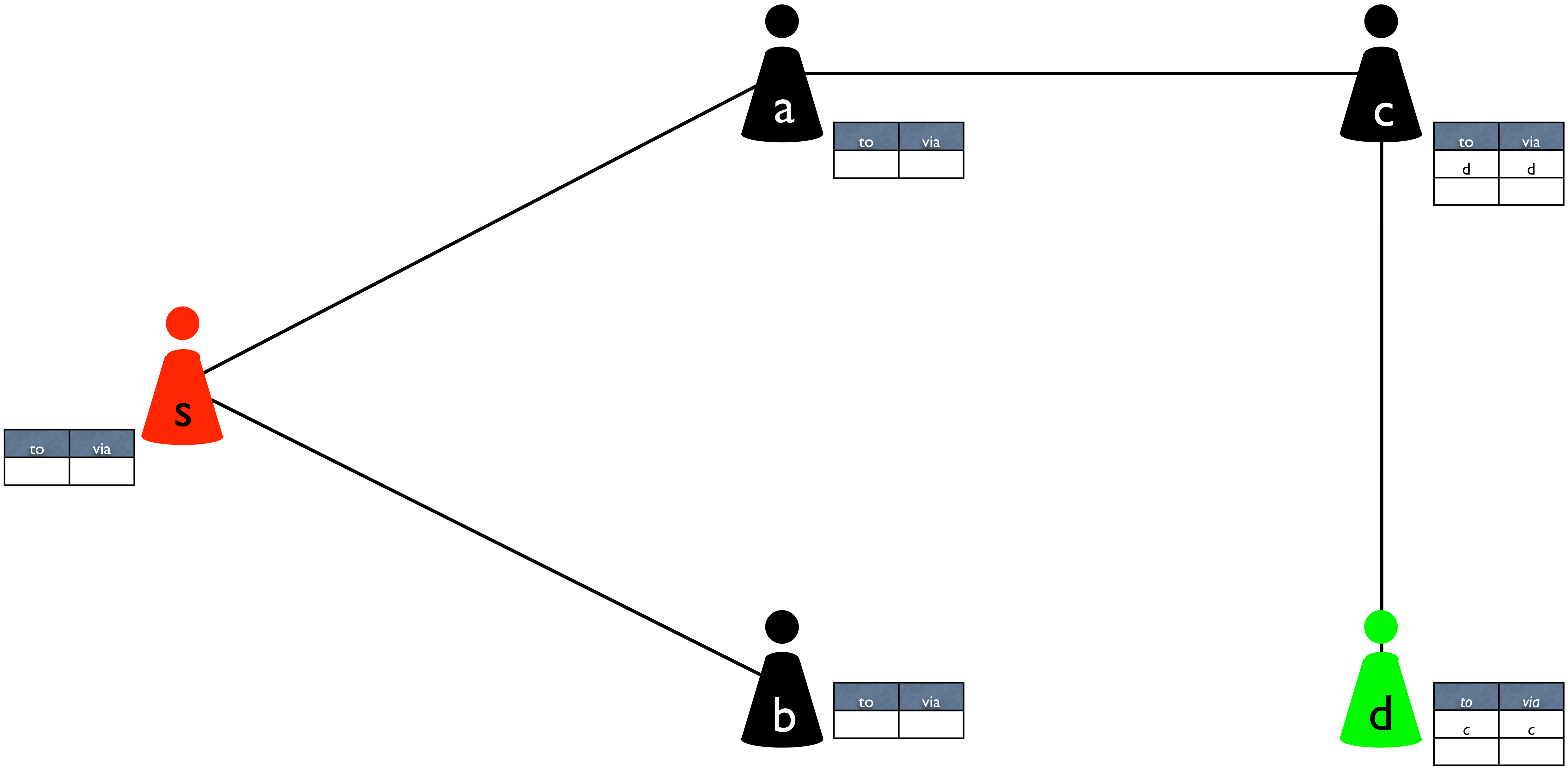


s is looking for a route to d

AODV – An Example



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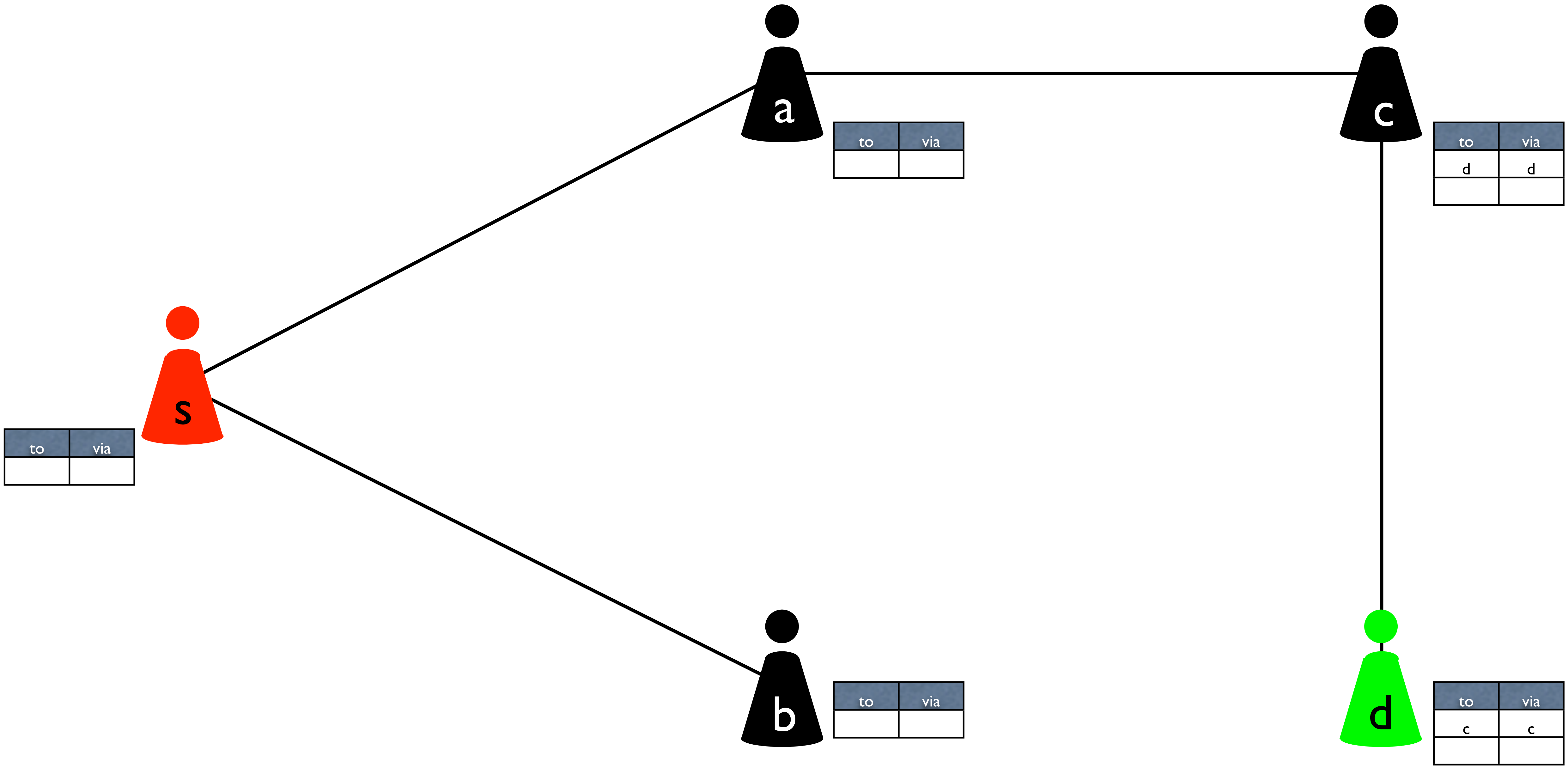
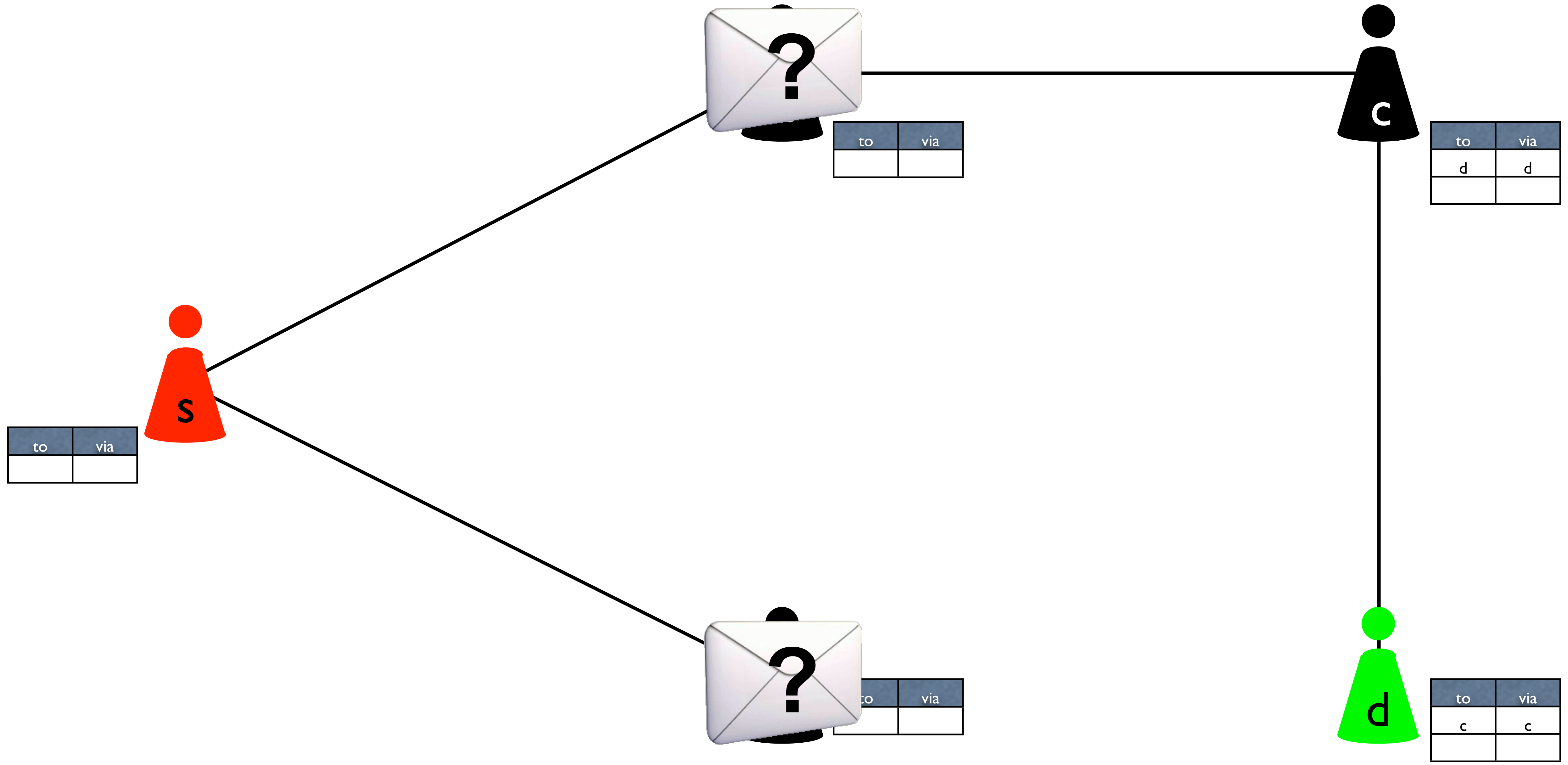


AODV – An Example



s broadcasts a route request

AODV – An Example

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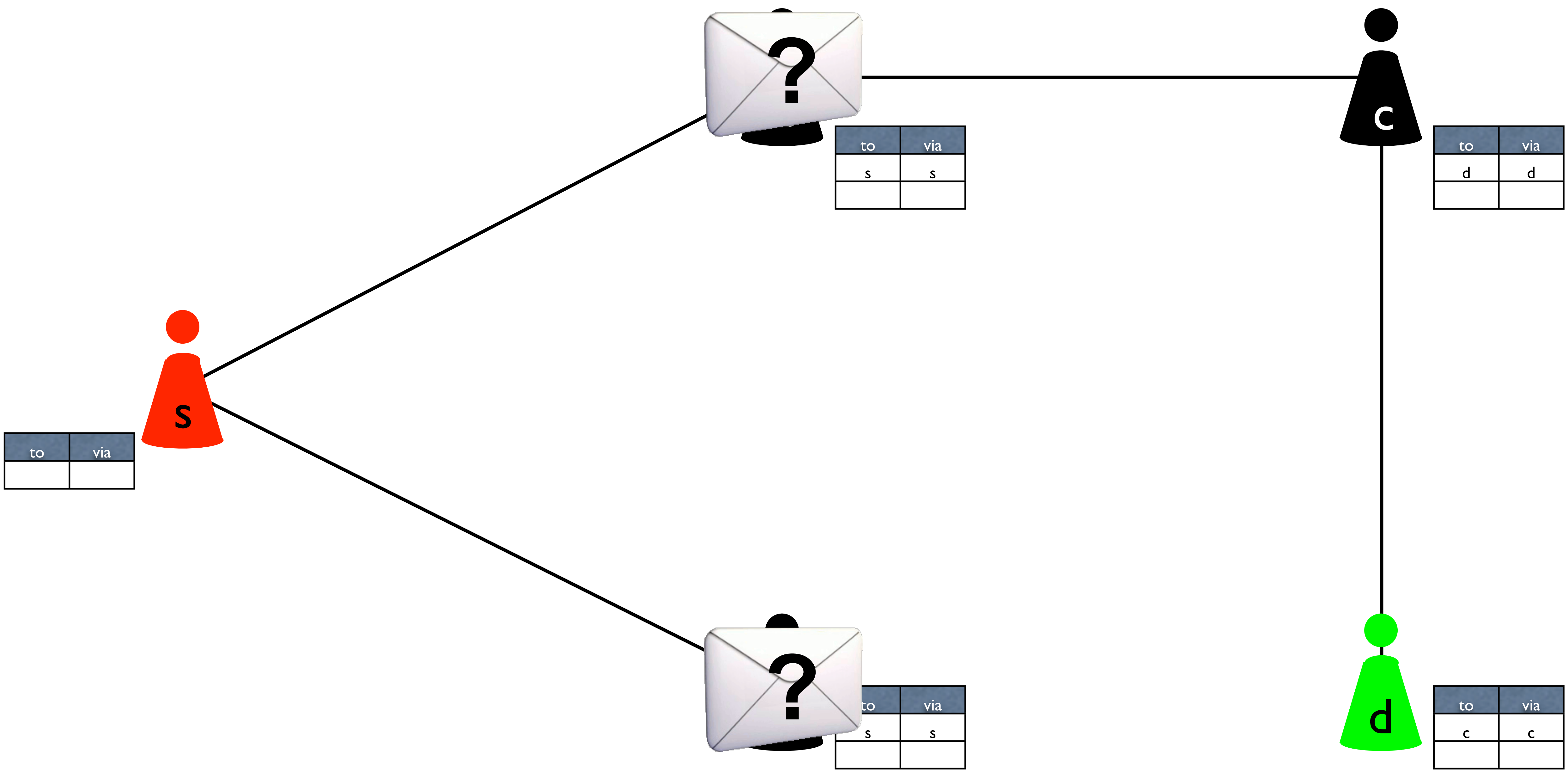

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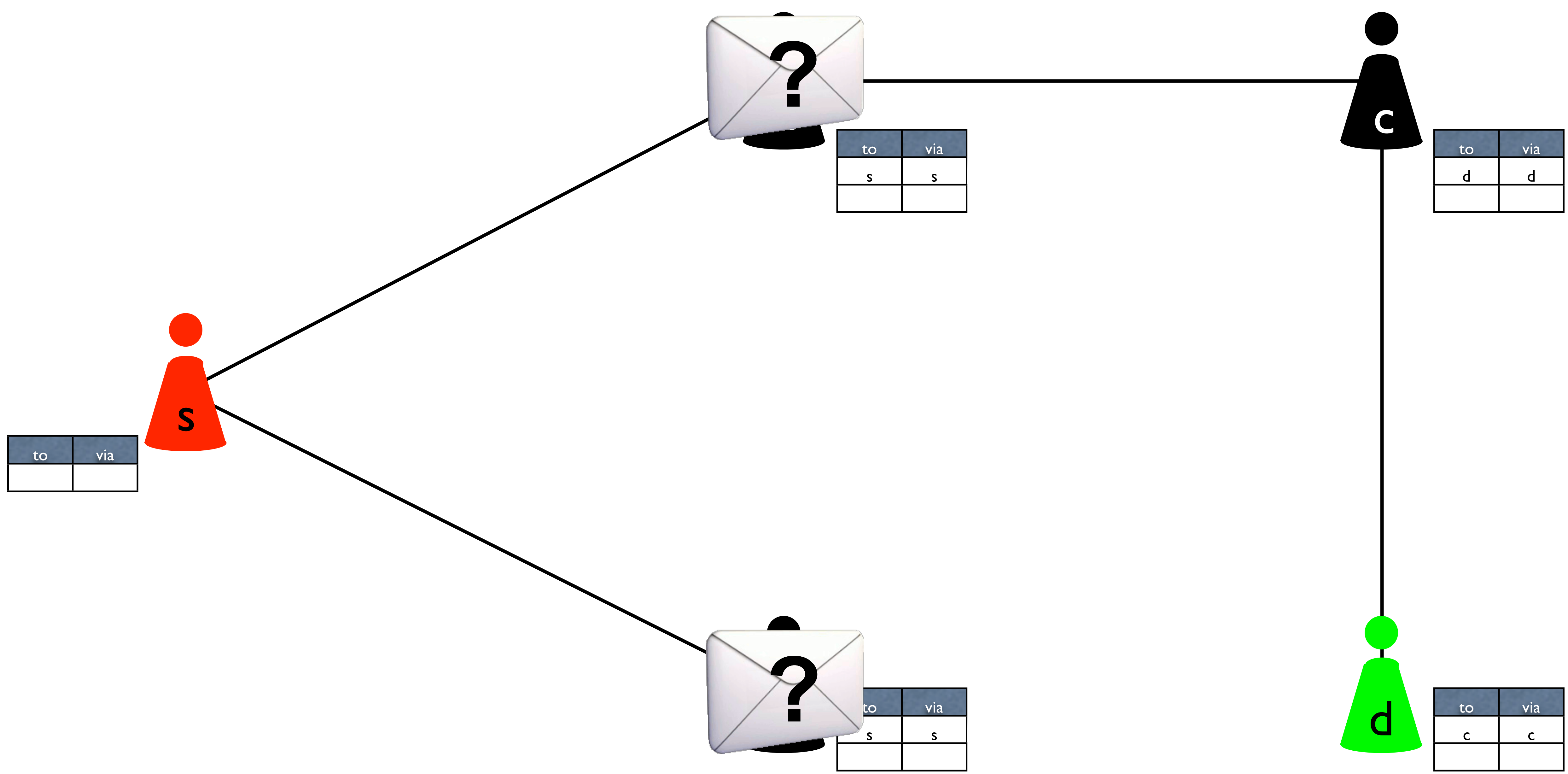

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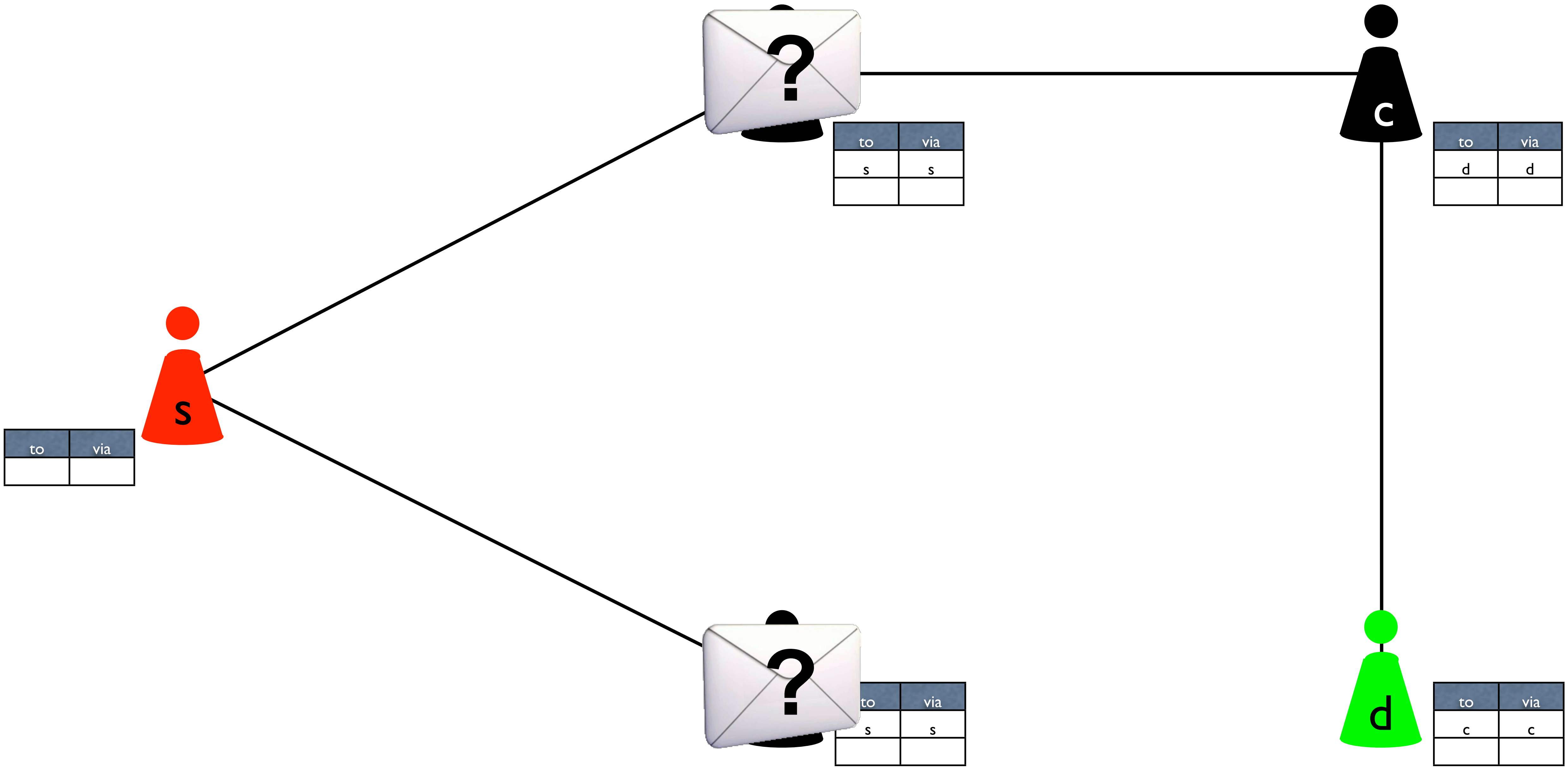

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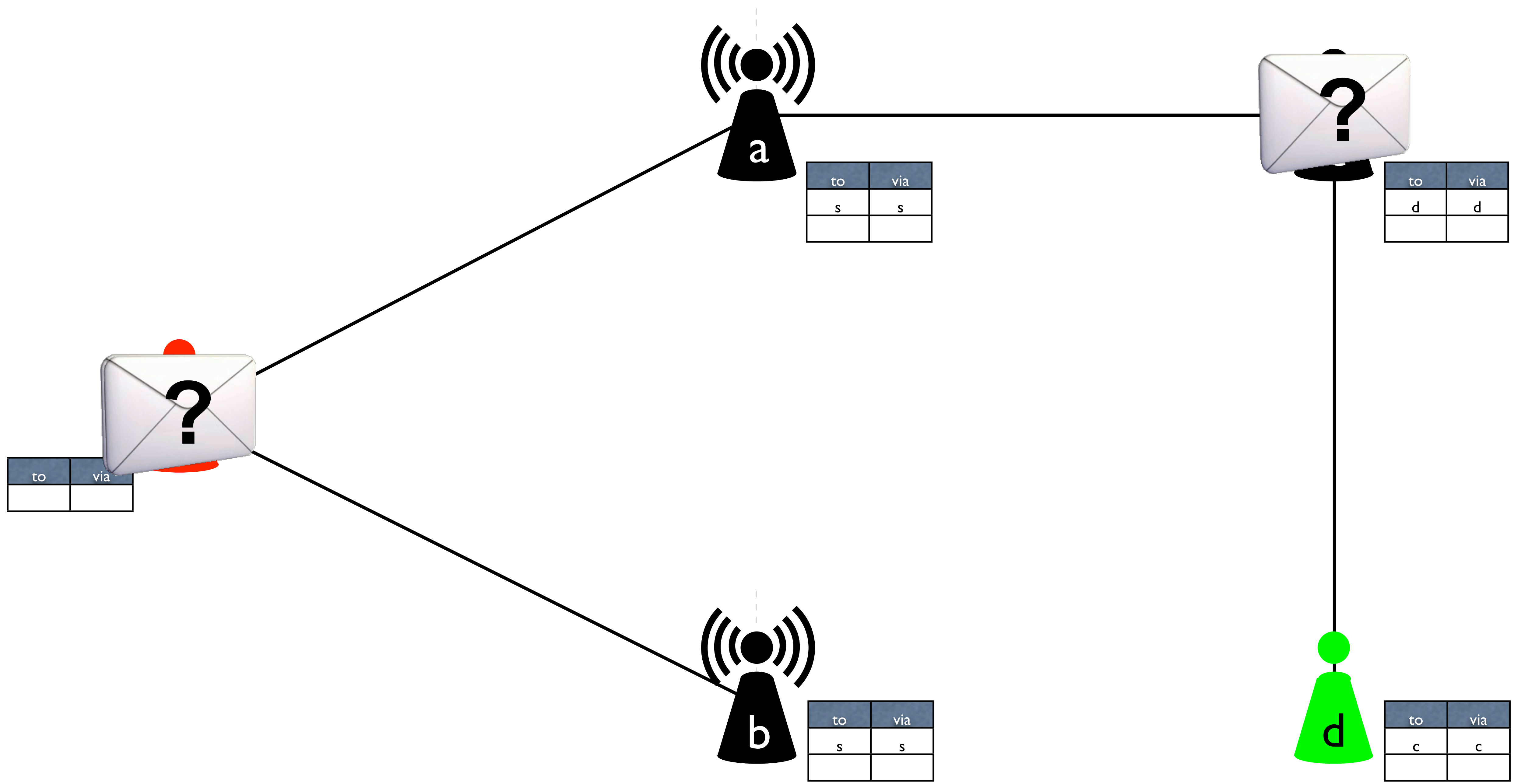
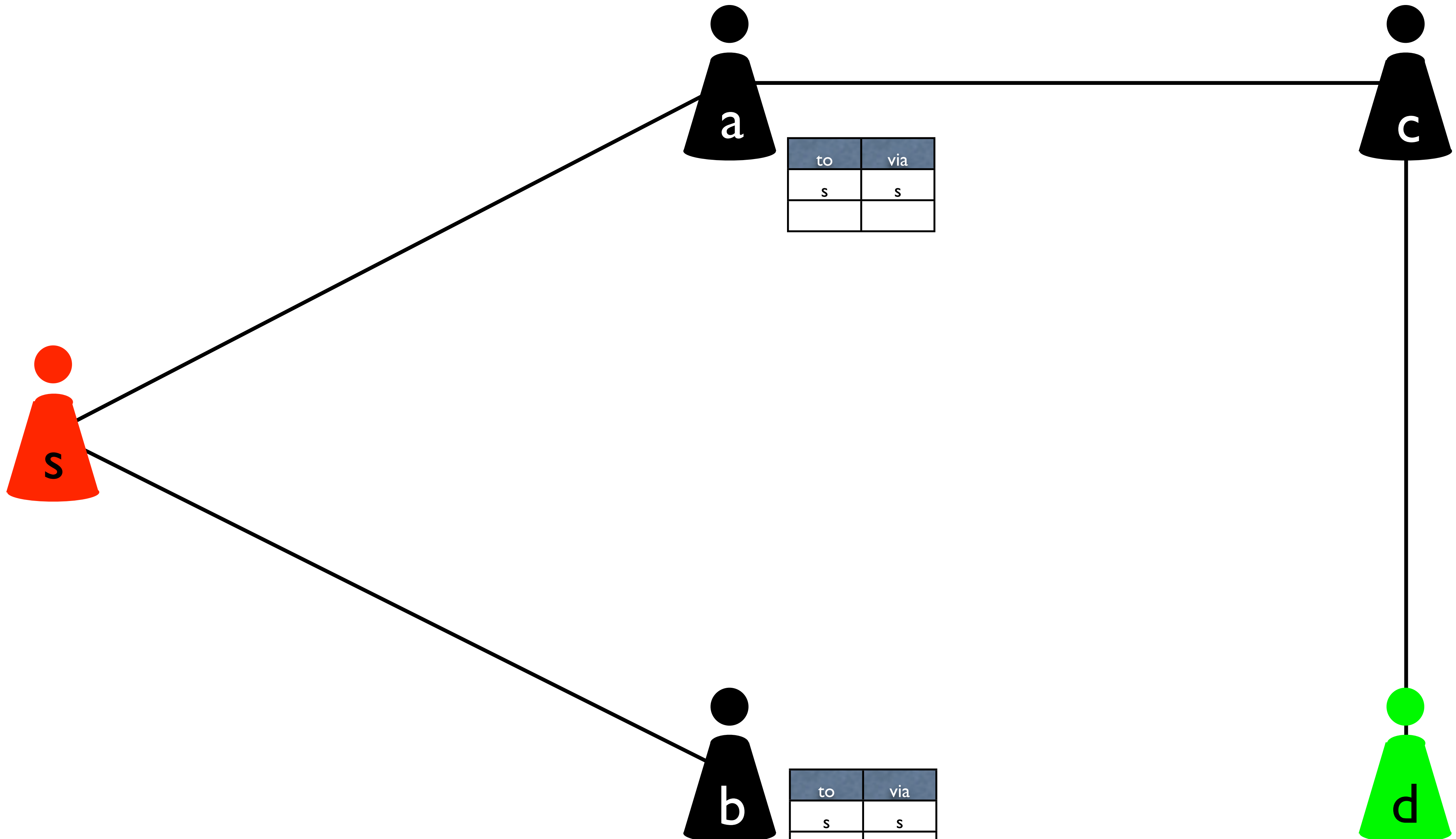


a,b forward the route request

AODV – An Example



to	via
a	a
b	b

to	via
s	s

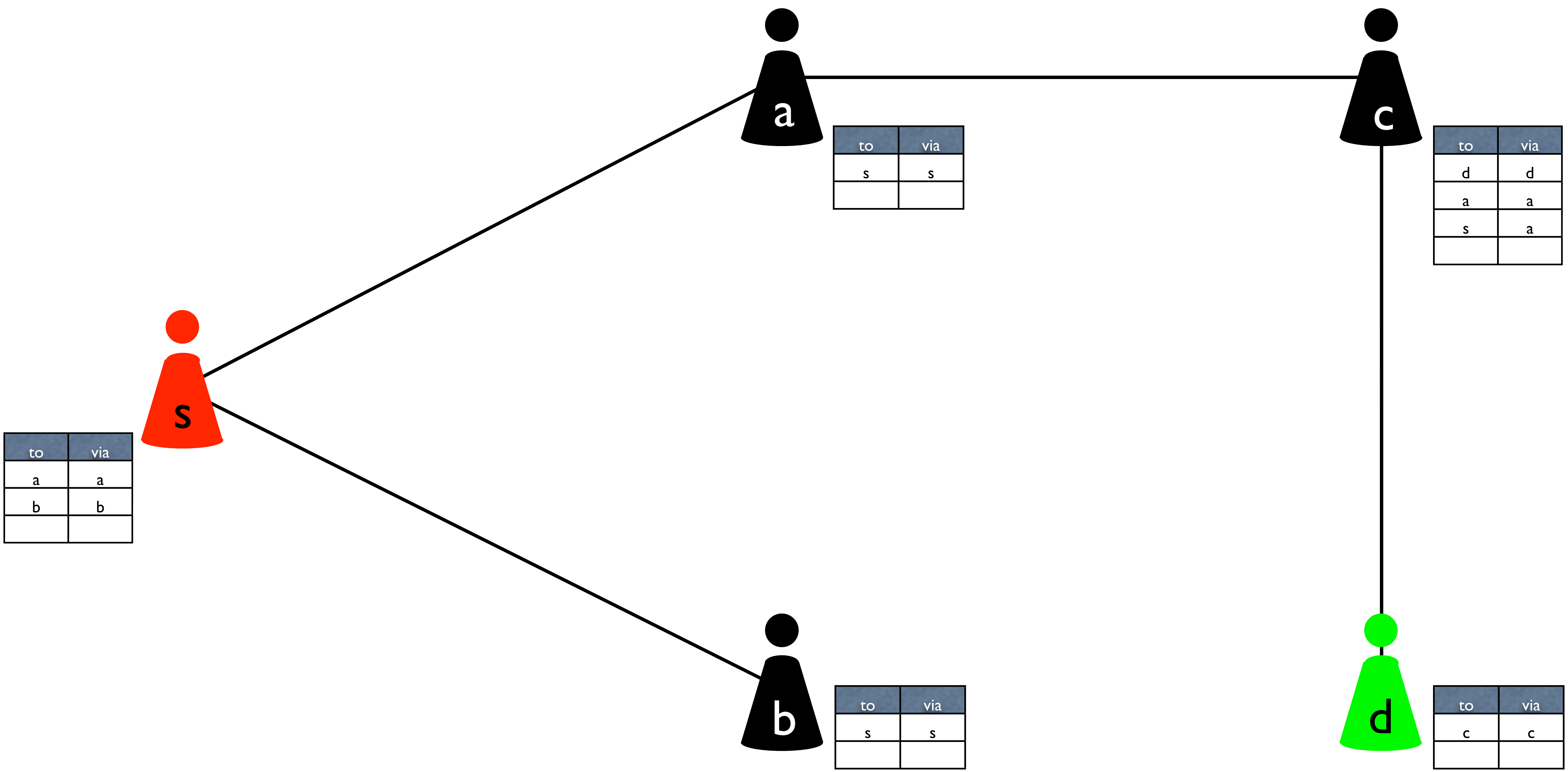
to	via
d	d
a	a
s	a

to	via
s	s

to	via
c	c

a,b forward the route request

AODV – An Example



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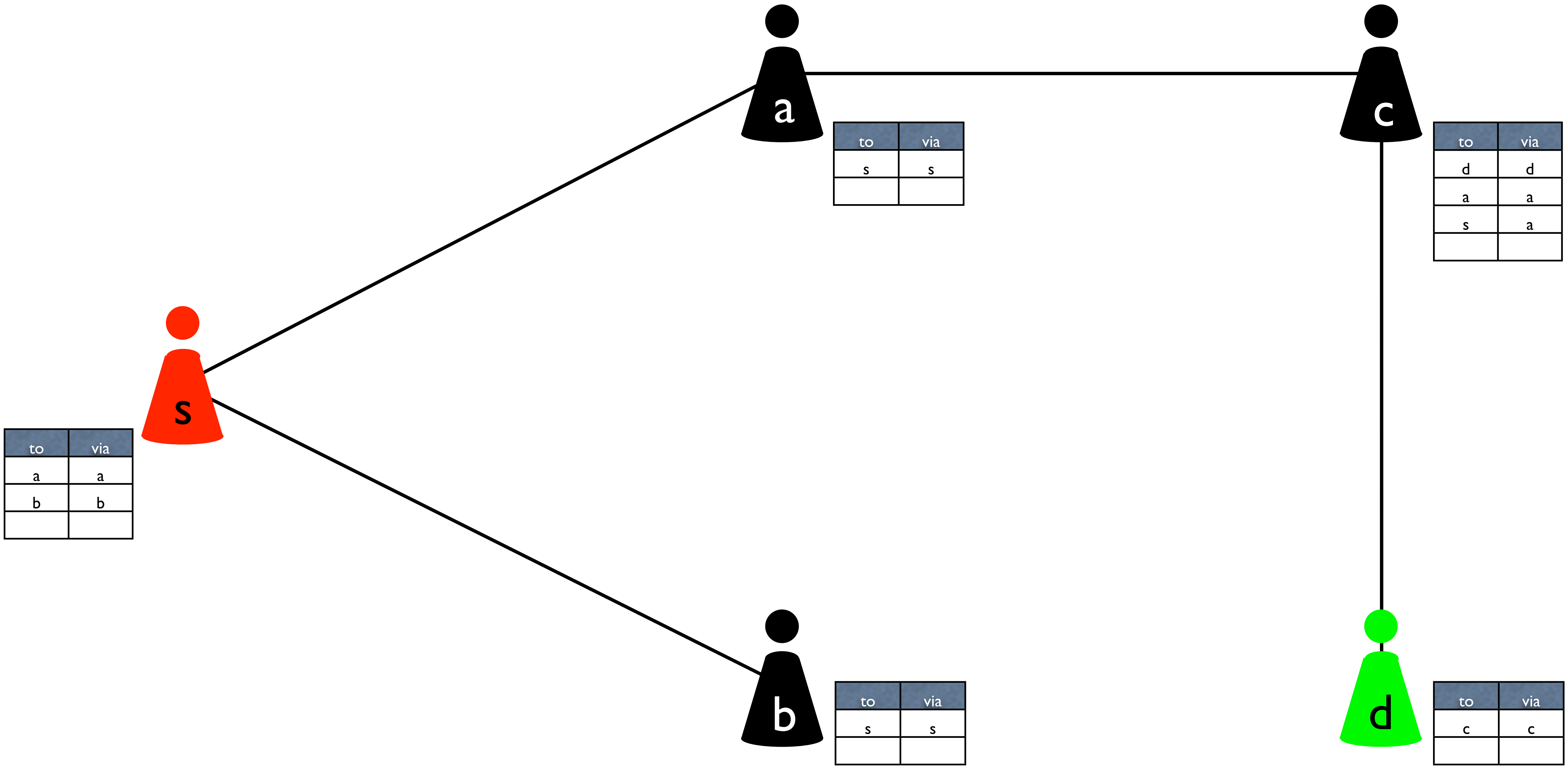

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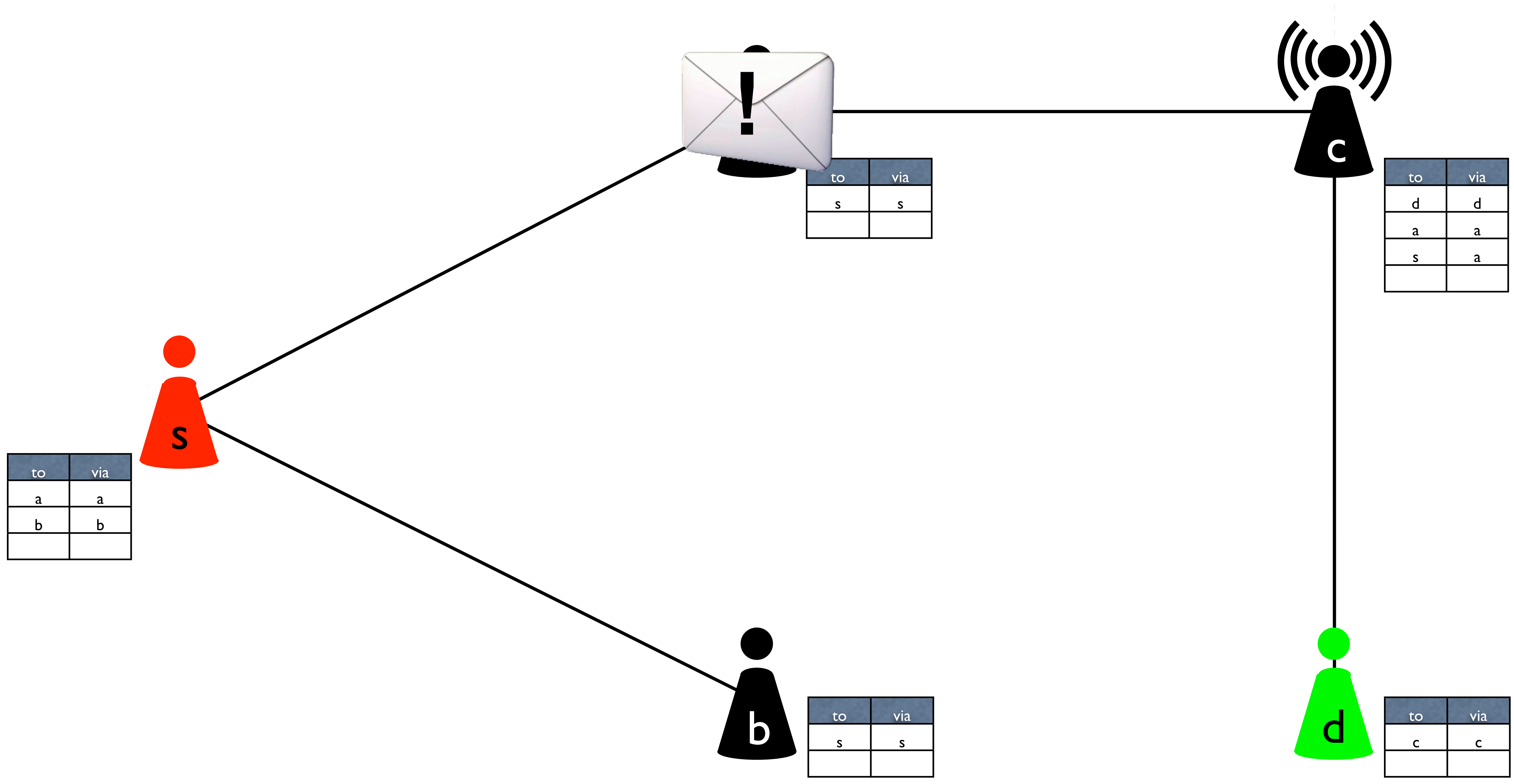
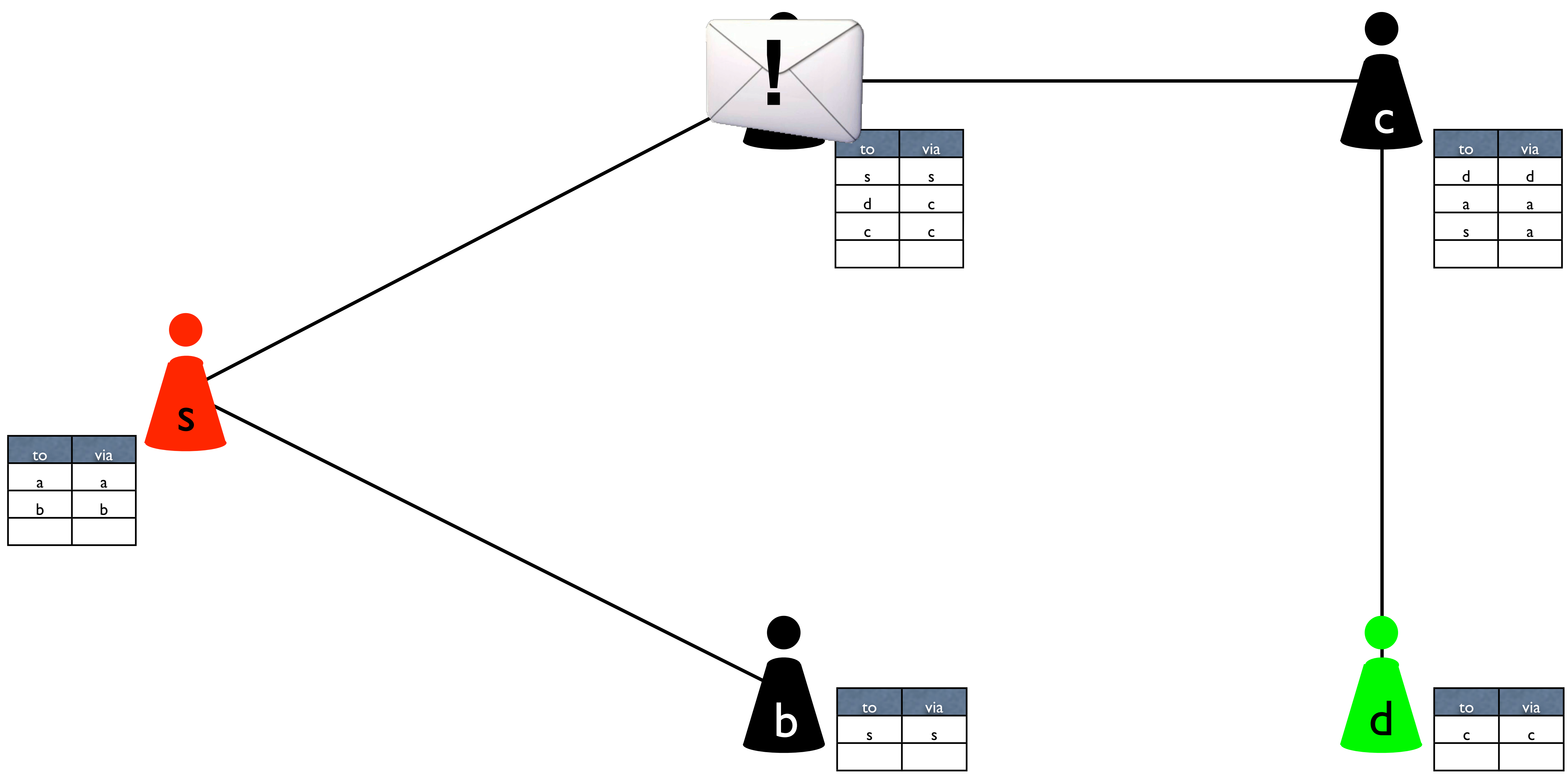


c has information about d
c answers route request and sends reply

AODV – An Example



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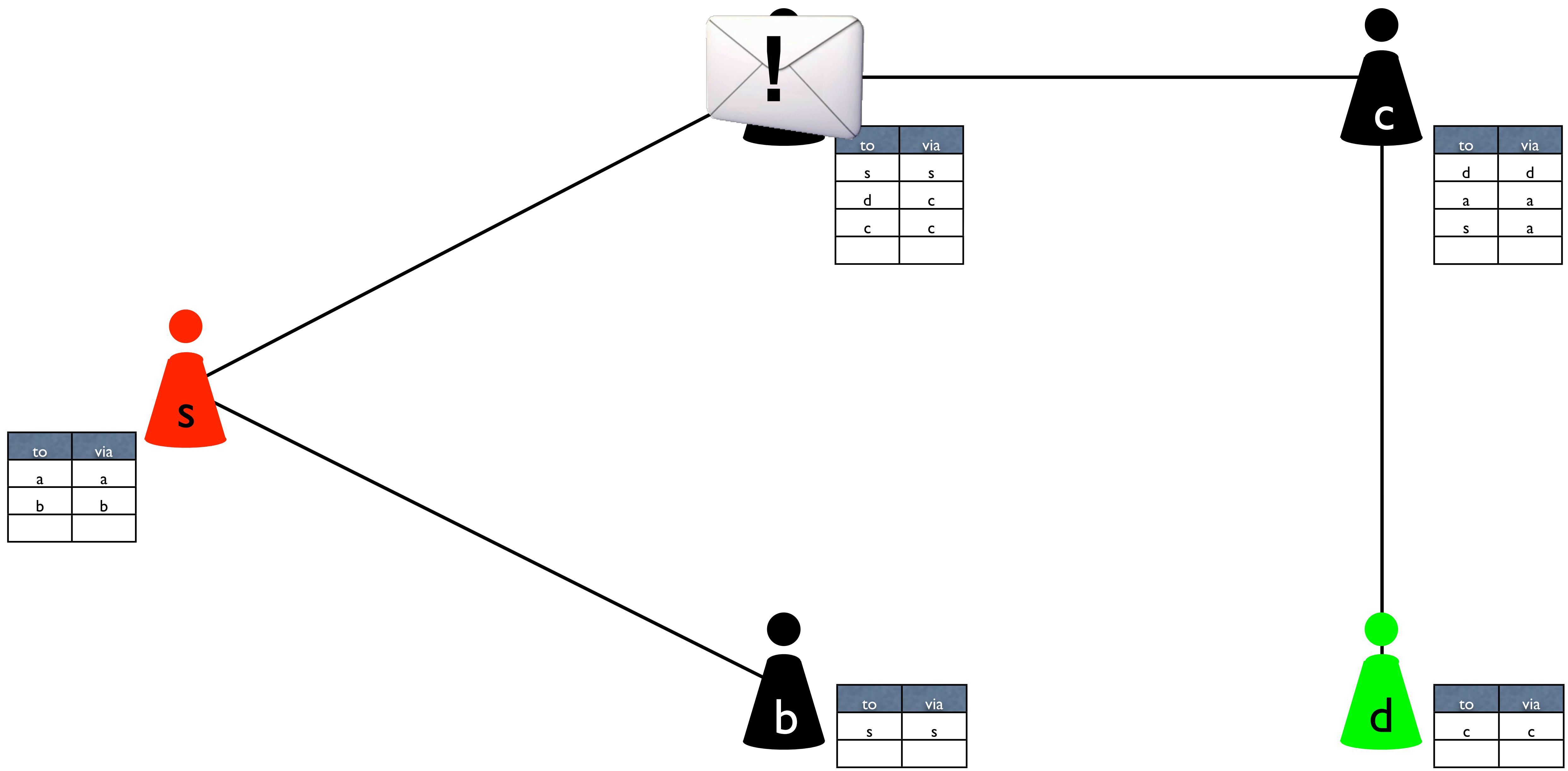

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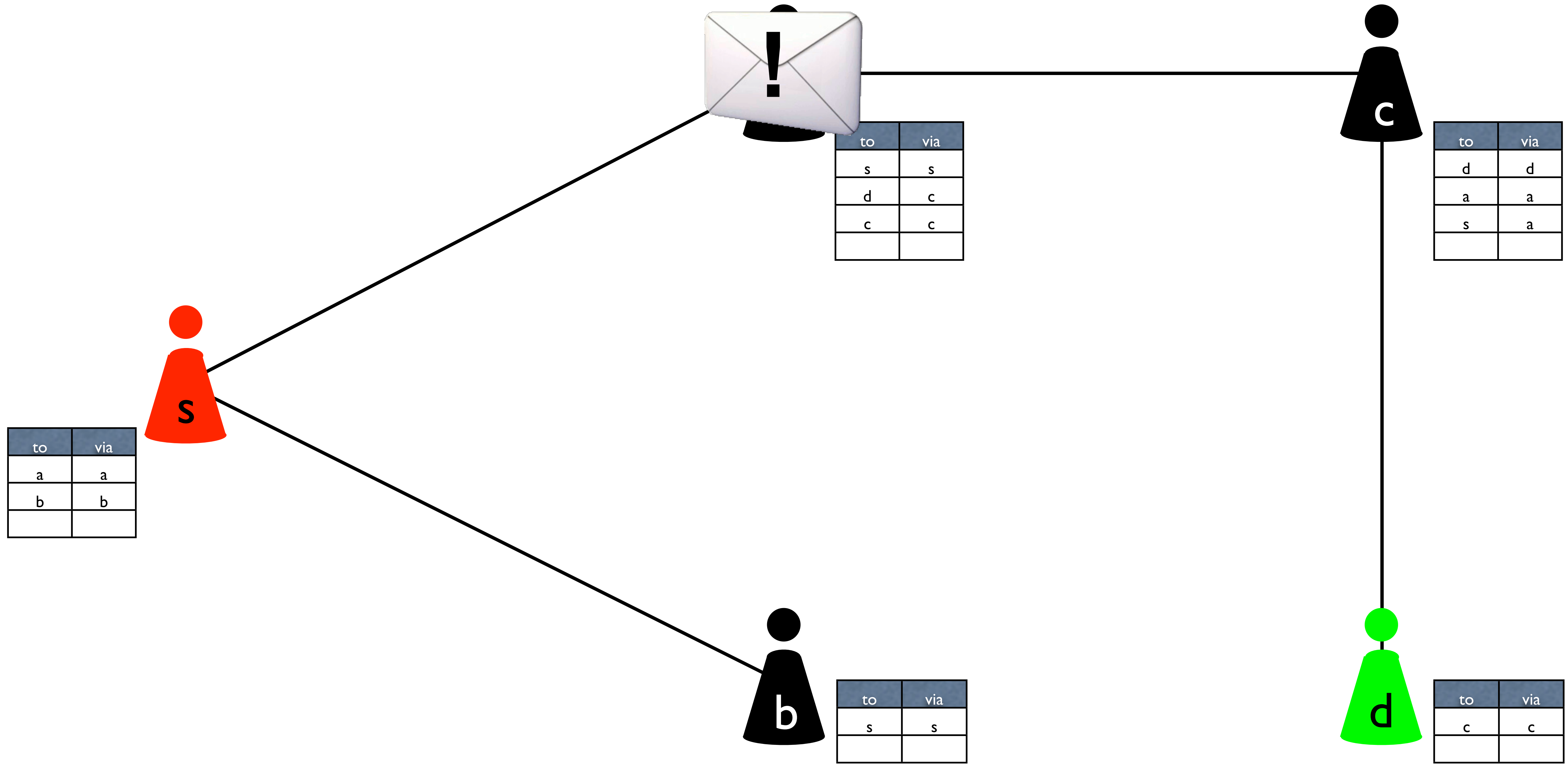

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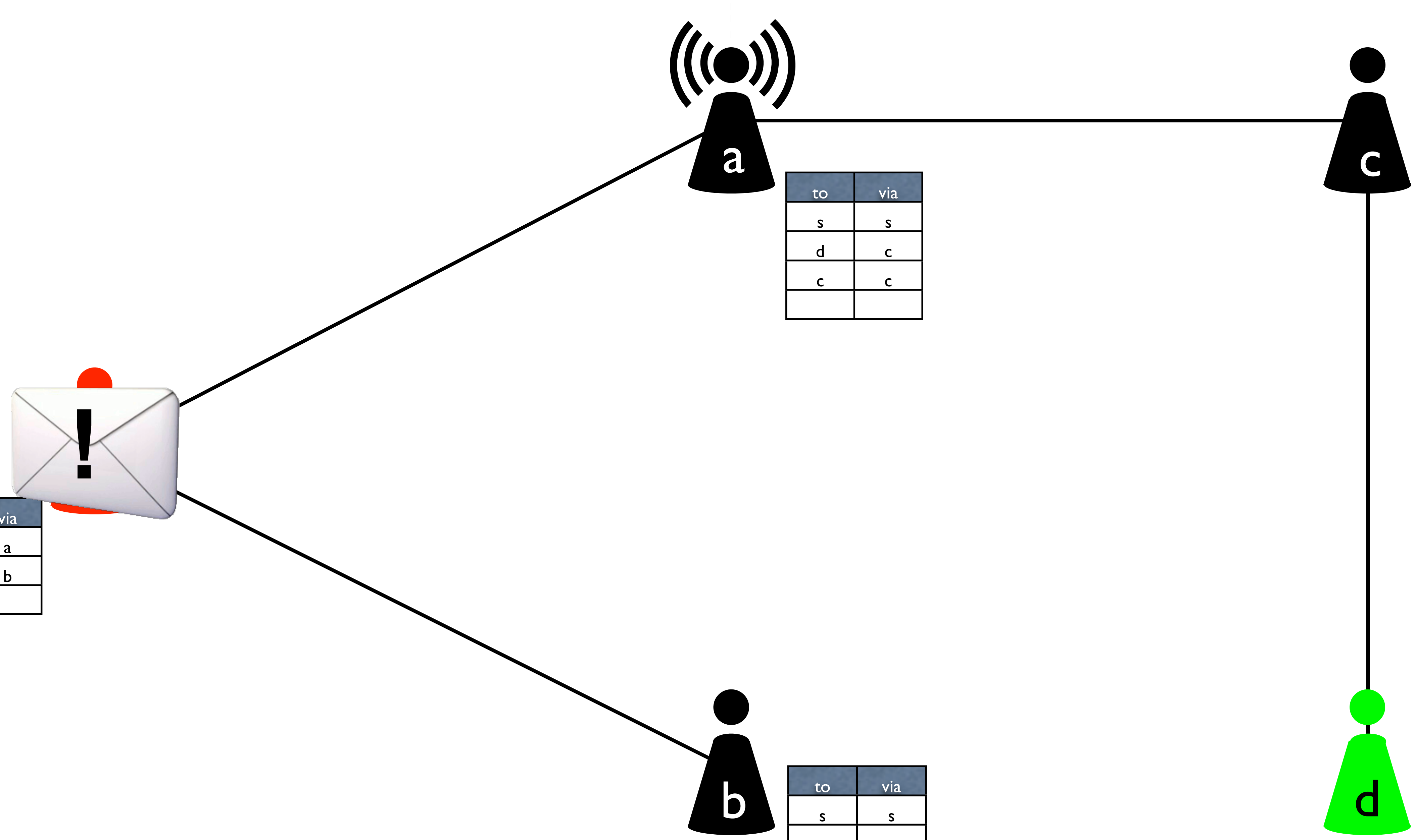


to	via
a	a
b	b

to	via
s	s
d	c
c	c

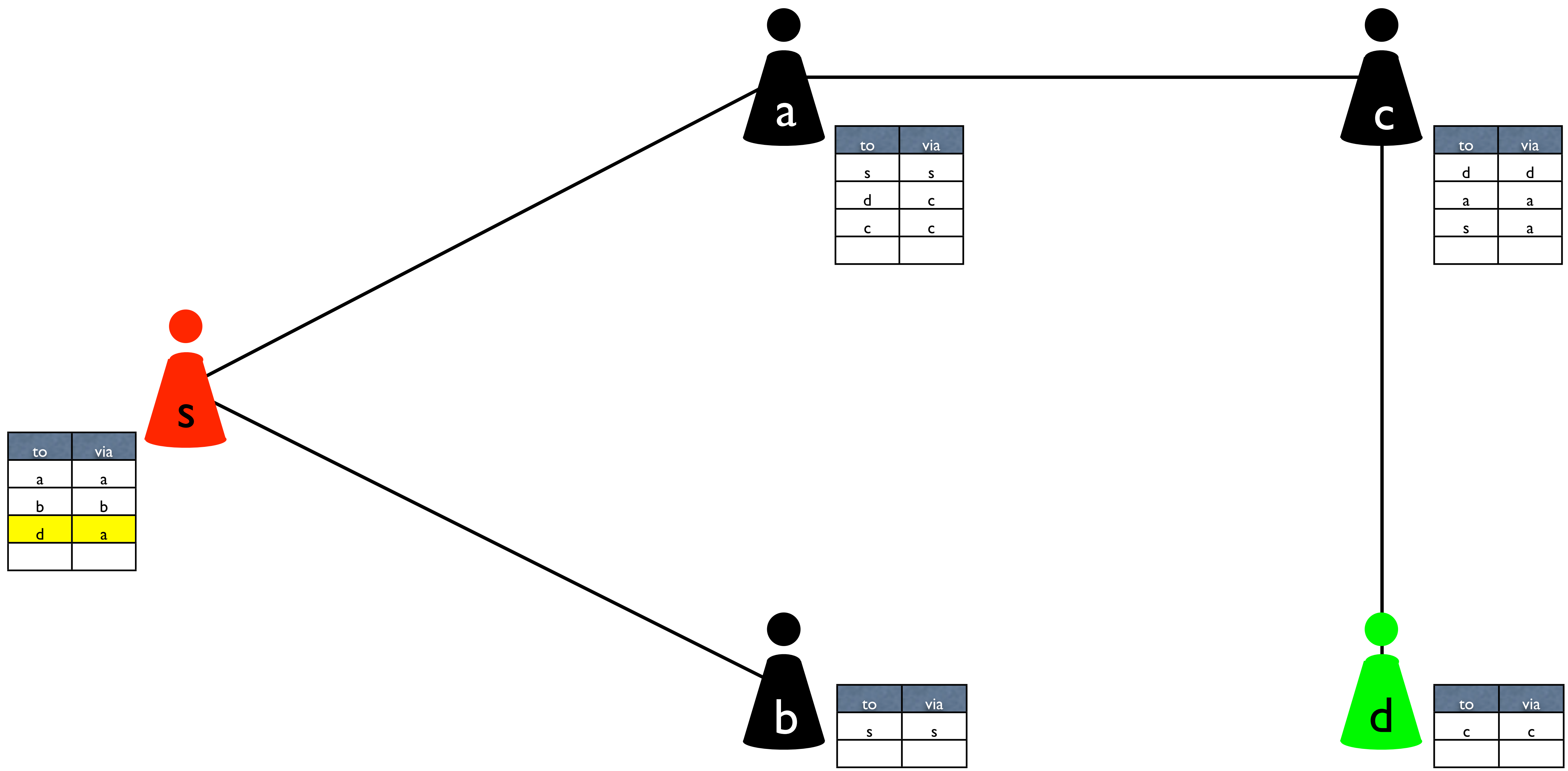
to	via
d	d
a	a
s	a

to	via
s	s

to	via
c	c

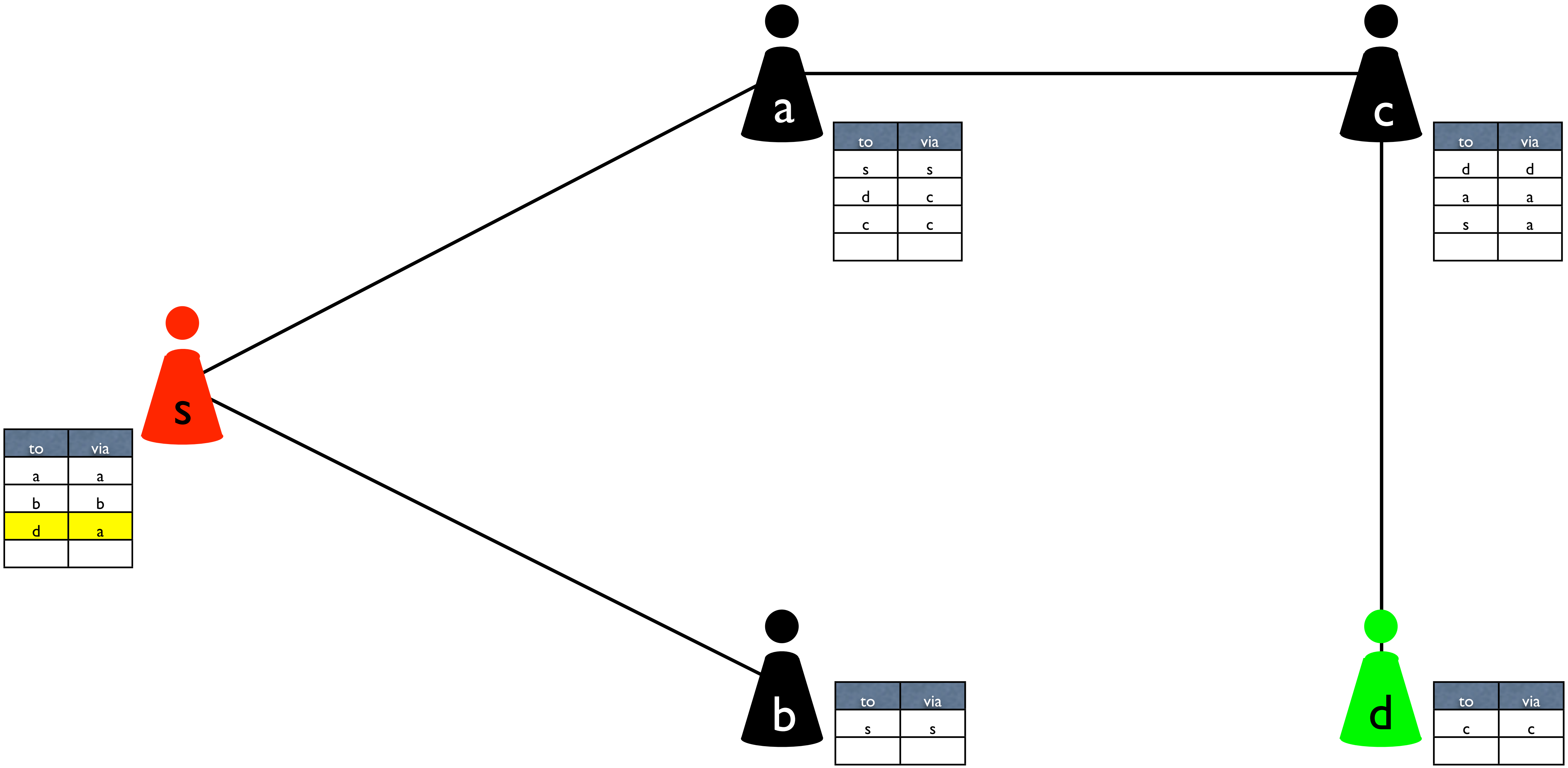
a forwards route reply

AODV - An Example



a forwards route reply

AODV – An Example



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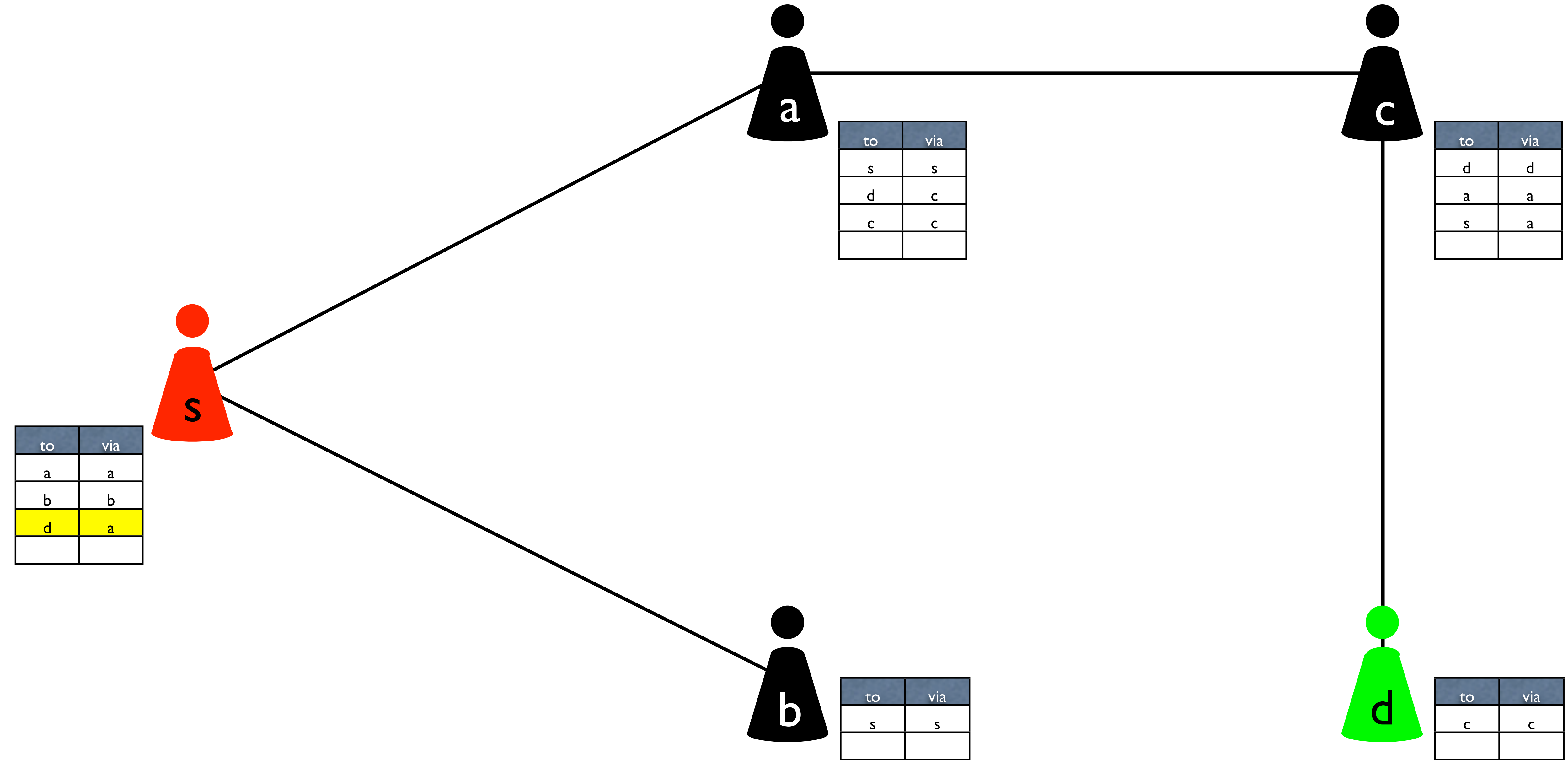

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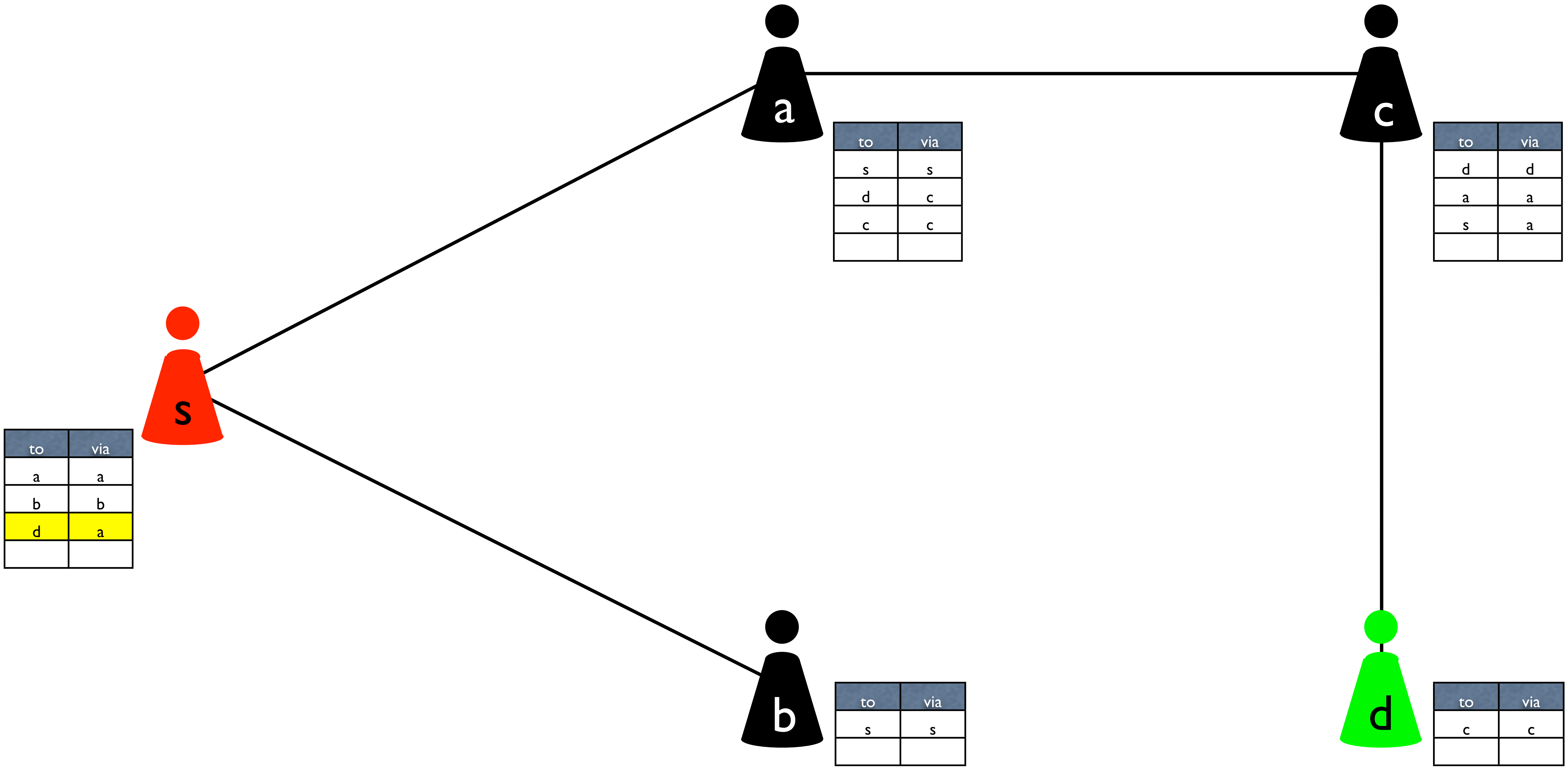
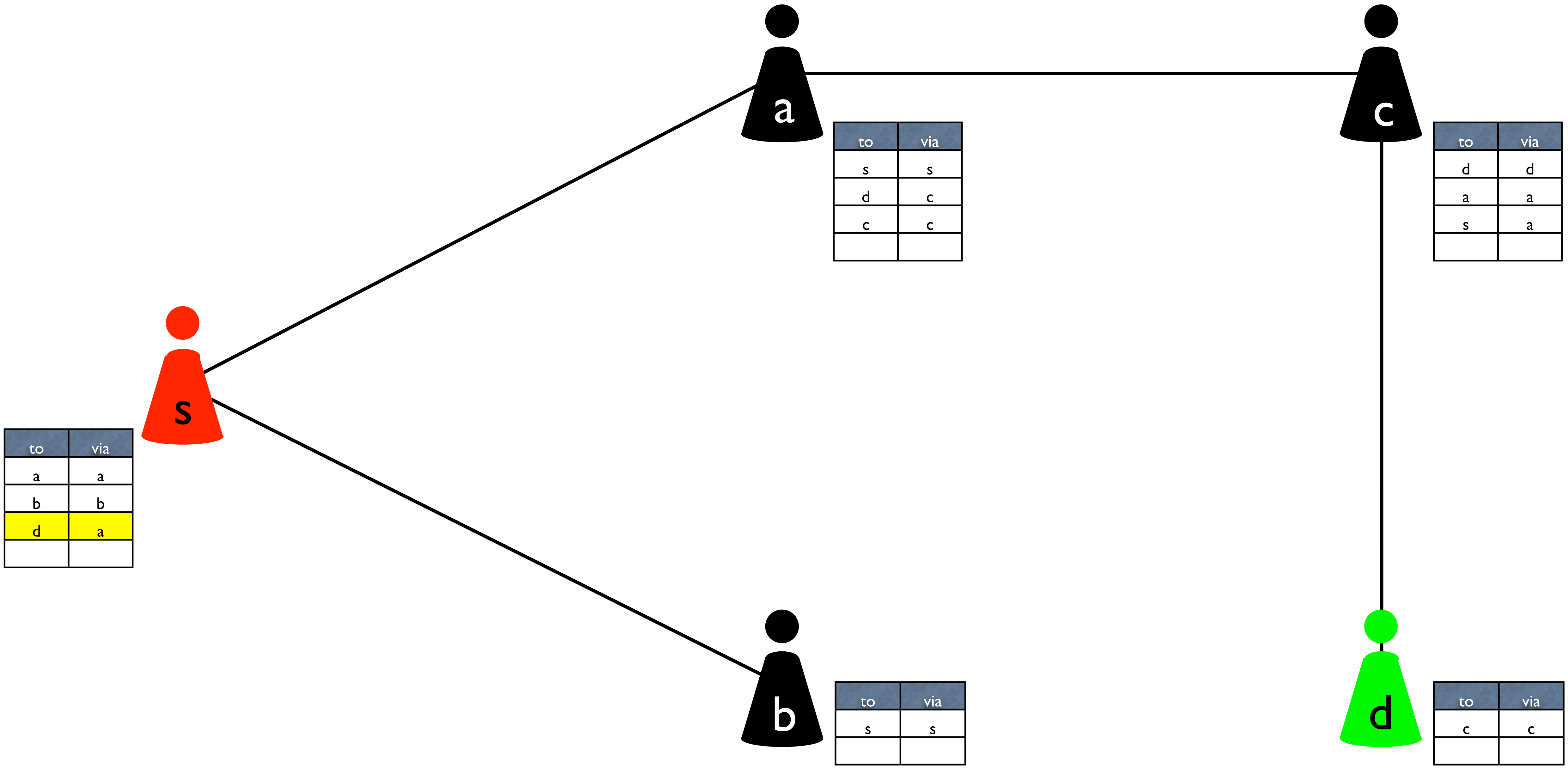


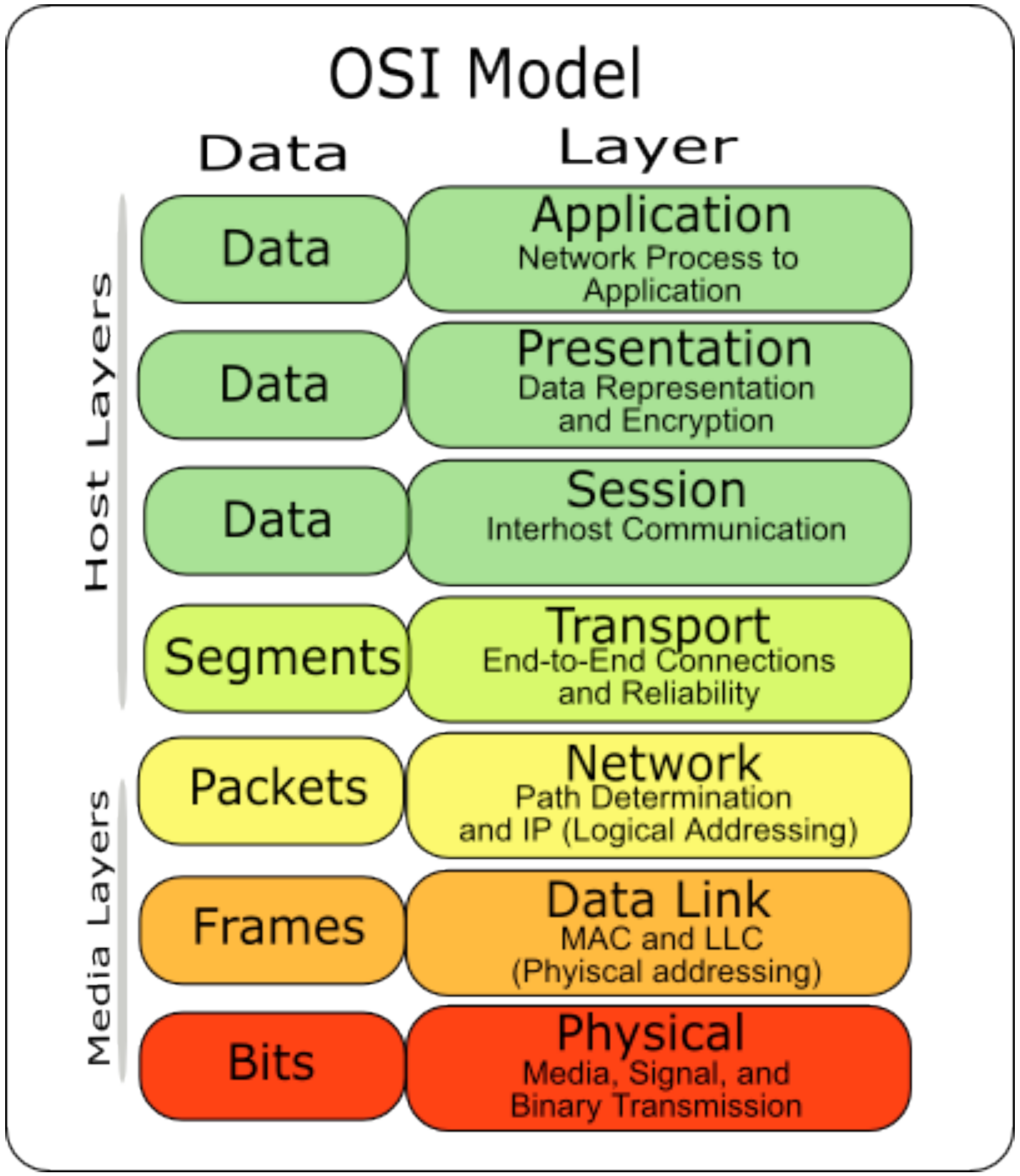
s has found a route to d

AODV – An Example



s has found a route to d

Different Network Layers



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Routing Protocols and Routing Tables

- Routing protocols
  - find (optimal) route
  - properties
    - loop freedom (no packet travels in loops)
    - route correctness (if a route is found, the route is valid)
    - route found (if a route exists, at least one route is found)
    - packet delivery

- Routing tables
  - data structure
  - belongs to client/router
  - lists destinations
  - sometimes metrics

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